**Problems to Introduction to Quantum Gravity I** WS 18/19

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### Problem 1: Variation of the Einstein-Hilbert action

Vary the Einstein-Hilbert action (with cosmological constant)

$$S_{\text{EH}} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^4x \sqrt{-g} \, (R - 2\Lambda) \tag{1}$$

w.r.t. the inverse metric $g^{\mu\nu}$ to obtain the vacuum Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0. \tag{2}$$

Hint: Show first that $\delta R_{\mu\nu}{}^{\rho}{}_{\sigma} = \nabla_\mu \left(\delta\Gamma^{\rho}_{\nu\sigma}\right) - \nabla_\nu \left(\delta\Gamma^{\rho}_{\mu\sigma}\right)$ and neglect boundary terms.

**Bonus question:**
Within the Riemann tensor, substitute the Christoffel symbols $\Gamma^{\rho}_{\mu\nu}$ by an arbitrary connection $A^{\rho}_{\mu\nu}$ that can be varied independently of the metric. Compute and solve the equations of motion obtained from varying $A^{\rho}_{\mu\nu}$.