Problems to Introduction to Quantum Gravity I

WS 18/19

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Sheet 6 — Handout: 27.11.2018 — To present: 03.12.2018

Problem 1: Dirac observables for the parametrised harmonic oscillator

Consider the parametrised harmonic oscillator, given by the Hamiltonian

$$H = \lambda\gamma, \quad \gamma := \left(p_t + \frac{p^2}{2} + \frac{q^2}{2}\right) \approx 0 \tag{1}$$

and Poisson brackets

$$\{t, p_t\} = 1, \quad \{q, p\} = 1. \tag{2}$$

Compute a complete set of independent Dirac observables using the gauge unfixing projector. Extract the relations $q(t)$ and $p(t)$ from the Dirac observables.

Problem 2: Relativistic particle: First order formulation

Consider the action

$$S[x^\mu, e] = \int d\tau \left(\frac{1}{4}e^{-1}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\eta_{\mu\nu} - em^2\right) \tag{3}$$

a) Perform the canonical analysis and compare to the result from the lecture.

b) Show that the mass shell condition together with the definition of p_μ fixes e in agreement with the lecture's results.

c) Show that by solving the equations of motion for the auxiliary field e, we obtain the action given in the lecture.

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Problem 3: BONUS: Homogeneous Lagrangians

Let $\mathcal{L}$ be a Lagrangian which is homogeneous of degree one in the velocities:

$$\mathcal{L}(q^i, c\,\dot{q}^i) = c\,\mathcal{L}(q^i, \dot{q}^i). \tag{4}$$

a) Show that the Hamiltonian is weakly zero and that there is at least one primary constraint.

b) Show that the Hamiltonian equations of motion are form-invariant w.r.t. different choices of time.

c) Given $\mathcal{L}$, construct a new Lagrangian $\mathcal{L}'$ which defines the same action principle as $\mathcal{L}$, but using velocities computed w.r.t. to a different time τ.