

Problem 1: Proca field

Consider the Lagrangian

$$L = \int d^3x \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu \right) \quad (1)$$

describing a massive spin 1 field A_μ , where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Perform the canonical analysis and show that there are 3 physical degrees of freedom per point, as opposed to 2 in the Maxwell case where $m = 0$.

Problem 2: Bonus exercise: Henneaux & Teitelboim: problem 1.26

Extend the analysis of Dirac to higher-order Lagrangians depending on the accelerations $\ddot{q}, \ddot{\dot{q}}, \dots$ up to order k .

Hint: Treat the velocities and the accelerations as independent variables by means of Lagrange multipliers in order to get a Lagrangian that contains only first-order time derivatives. The extra added variables can be eliminated by means of their own equations and are thus auxiliary fields.