Problems to Introduction to Quantum Gravity I WS 18/19

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Sheet 2 — Handout: 30.10.2018 — To present: 05.11.2018

Problem 1: Inconsistent systems

Find a Lagrangian which results in an inconsistent Hamiltonian system. What was wrong with the original Lagrangian?

Problem 2: Algebraic structure of gauge transformations

a) Show that the poisson bracket of two first class functions is again first class.

b) Apply two successive gauge transformations and subtract from the result their application in reverse order. Show that the resulting transformation is generated by the Poisson bracket of the two generators.

Hint: Use the properties of the Poisson bracket discussed in the lecture.

Problem 3: Hennaux & Teitelboim: problem 1.9

Show that if the constraints

$$p_a + K_a(q^i, p_j, q^a) \approx 0 \tag{1}$$

($a = 1, \ldots, A$; $i, j = A+1, \ldots, N$) are first class, and if there is no other constraint in the theory, then the Poisson bracket $[p_a + K_a, p_b + K_b]$ vanishes $s$trongly.

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Problem 4: Henneaux & Teitelboim: problem 1.10

Consider a system with constraints

$$\pi_k = 0, \quad \phi_\alpha(q^i, p_i, \lambda^k, \pi_k) = 0 \tag{2}$$

where (q^i, p_i) and (λ^k, p_k) are canonically conjugate pairs. Assume that the constraints are all first class.

- **a)** Show that one can redefine the constraints to $(\pi_k = 0,\ \phi_\alpha = 0) \to (\pi_k = 0,\ \psi_\alpha = 0)$ in such a way that the new constraint functions ψ_α do not depend on either π_k or λ_k, so that $\psi_\alpha = \psi_\alpha(q^i, p_i)$.
- **b)** Proof that any first class function is weakly equal to a function of q^i and p_i only.

Hints: (i) Proof that one can eliminate the π_k-dependence from ϕ_α. (ii) Show next that the resulting constraint cannot depend on λ_k.