

# Mental Probability Logic—A Competence Model of Human Rationality

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## 1. Introduction

### Question

What is an appropriate normative standard of reference for evaluating the rationality of human reasoning?

### Ideas

- *Problem*: Classical logic is too strict and monotone.
- Traditional theories of human reasoning use wrong rationality norms!
- *Nonmonotonicity*: People withdraw conclusions.
- Soft version of *if-then*, that allow for *exceptions*:

$$\text{If } A, \text{ normally } B \quad \Leftrightarrow \quad P(B|A) > .5$$

## 2. Human Nonmonotonic Reasoning

### Introduction

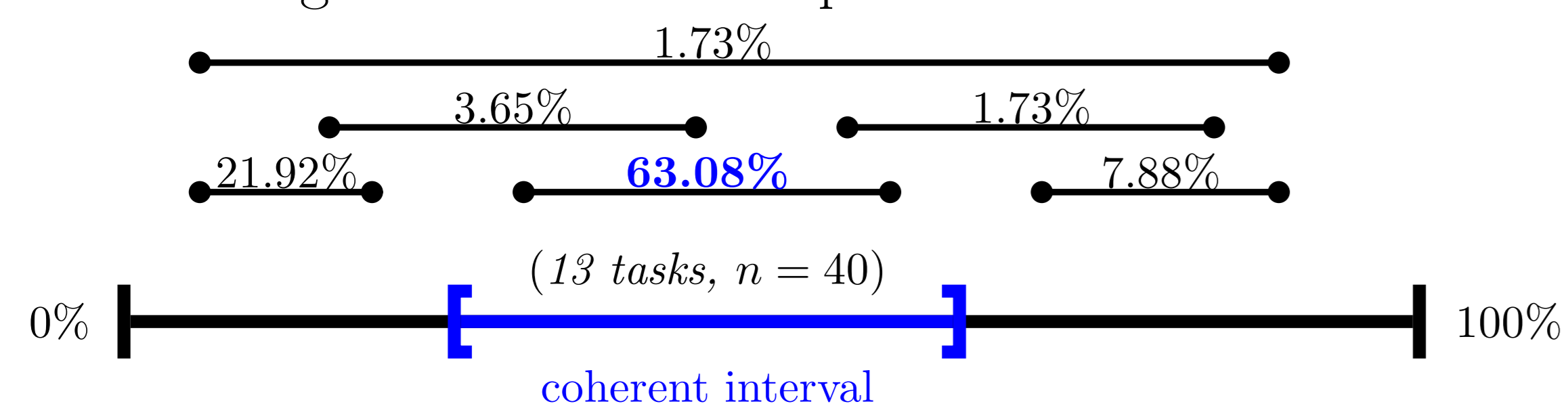
- SYSTEM P is a set of basic rationality postulates every system of nonmonotonic reasoning should satisfy (Kraus, Lehmann & Magidor, 1990).
- SYSTEM P combines logic and probability by inference rules, like  $P(B|A)=x, P(C|A)=y \Rightarrow P(B\&C|A) \in [\max(0, x+y-1), \min(x, y)]$ .

### Method (Pfeifer & Kleiter, 2005a, 2006)

You know about a tourist party from Alsace:  
70% of the tourist party speak *French*.  
60% of the tourist party speak *German*.  
How many of the tourist party speak *both French and German*?

### Results

Six possible categories of interval responses:



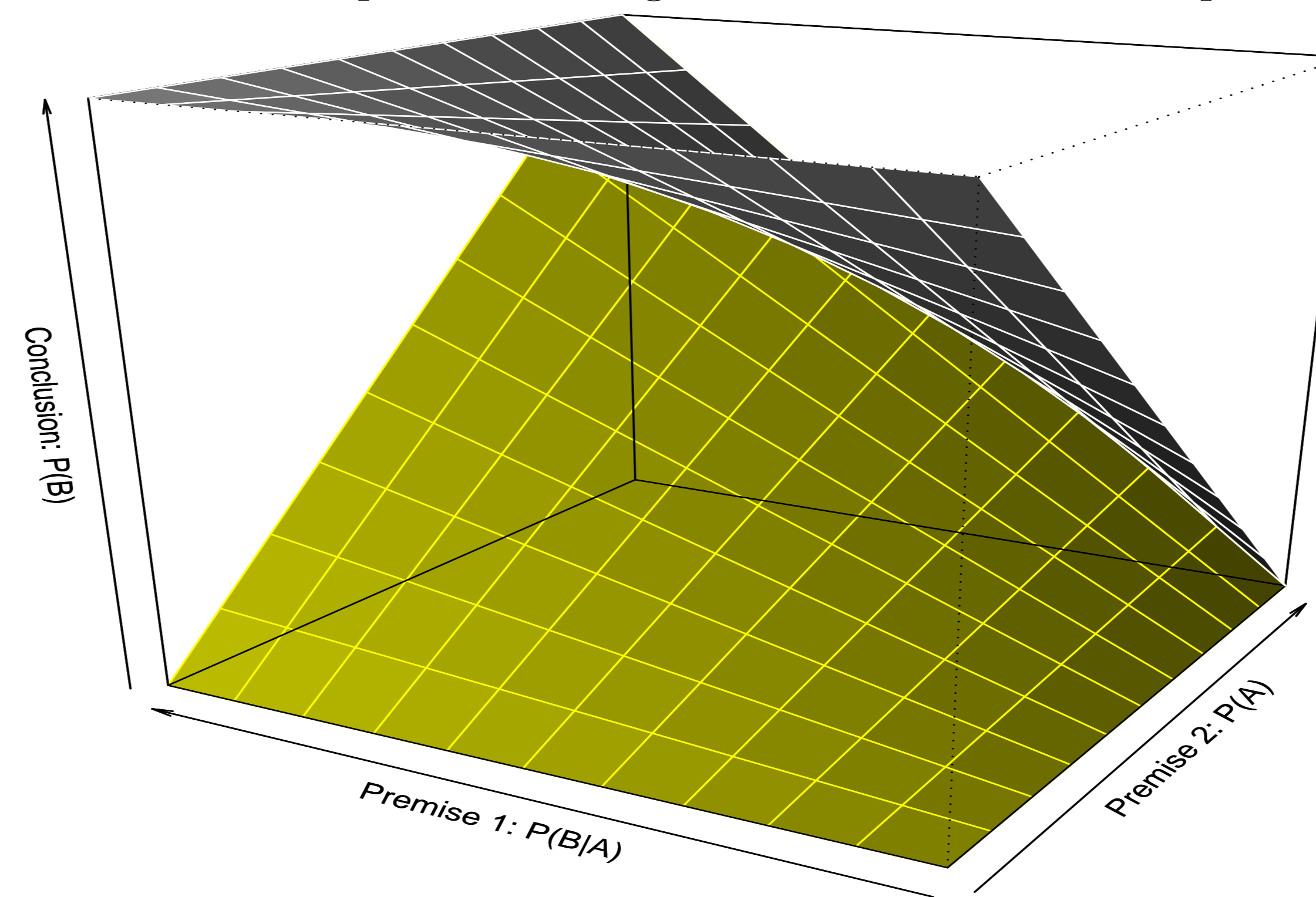
### References

- Pfeifer, N. (2006). Contemporary syllogistics: Comparative and quantitative syllogisms. In G. Kreuzbauer & G. J. W. Dorn (Eds.), *Argumentation in Theorie und Praxis: Philosophie und Didaktik des Argumentierens* (p. 57-71). Wien: LIT.
- Pfeifer, N., & Kleiter, G. D. (2005a). Coherence and nonmonotonicity in human reasoning. *Synthese*, 146(1-2), 93-109.
- Pfeifer, N., & Kleiter, G. D. (2005b). Inference in conditional probability logic. In *Proceedings of the 8<sup>th</sup> Czech-Japan seminar on data analysis and decision making under uncertainty* (p. 101-108). Trřest, Czech Republic.
- Pfeifer, N., & Kleiter, G. D. (2005c). Towards a mental probability logic. *Psychologica Belgica*, 45(1), 71-99.
- Pfeifer, N., & Kleiter, G. D. (2006). Is human reasoning about nonmonotonic conditionals probabilistically coherent? In *Proceedings of the 7<sup>th</sup> workshop on uncertainty processing*. Mikulov, Czech Republic: September 16-20<sup>th</sup>, 2006.

- **Nonmonotonic** inference rules are valid in SYSTEM P and are **endorsed** by the subjects: the majority infers *coherent* intervals.
- **Monotonic** inference rules are not valid in SYSTEM P and are **not endorsed**: the subjects infer wide and not informative intervals.

## 3. Mental Probability Logic (Pfeifer & Kleiter, 2005c, 2005b)

**Mental probability logic** is a competence theory about how humans interpret and reason about common sense conditionals. Common sense conditionals are interpreted as “high” coherent conditional probabilities.



**Figure:** coherent intervals of the probabilistic MODUS PONENS. Coherent lower probability bounds are in green. Coherent upper probability bounds are in grey.

- Conditional probability logic is the normative standard of reference.
- Conditional probability logic interprets inference rules of logic probabilistically, e.g., the MODUS PONENS:

	Logic	Conditional probability logic
Premise 1:	$A \rightarrow B$	$P(B A) = x$
Premise 2:	$A$	$P(A) = y$
Conclusion:	$B$	$P(B) \in [xy, 1 - y + xy]$
	(log. valid)	(prob. informative)

- There are logically valid inference rules that are probabilistically not informative: e.g., CONTRAPOSITION, TRANSITIVITY.
- There are logically not valid inference rules that are probabilistically informative: e.g., DENYING THE ANTECEDENT.
- If subjects infer coherent intervals in the DENYING THE ANTECEDENT, they are *not* irrational—as usually claimed in classical

logic based theories of reasoning.

## 4. Syllogistic Reasoning (Pfeifer, 2006)

Originated by Aristotle (\*384, †322), syllogisms are special argument forms with two premises and one conclusion. Since Störing (1908), syllogisms are prototypical reasoning tasks in psychology.

### Problem

The universal quantifier is too strong, while the existential quantifier is too weak. Thus, classical syllogistics is not an appropriate rationality norm.

### Solution

Replace the classical quantifiers by “realistic” quantifiers (“Most”, “Almost all”, “All, except  $x$ ”, “ $x/n$ ”), e.g.,

$$\begin{array}{l} \text{Most birds have wings.} \quad |\{B \cap W\}| > |\{B \cap \bar{W}\}| \\ \text{Most birds have feathers.} \quad |\{B \cap F\}| > |\{B \cap \bar{F}\}| \\ \text{At least one thing has feathers and wings.} \quad |\{F \cap W\}| > 0 \end{array}$$

... reasoning about **frequencies**.

## 5. Concluding Remarks

- Behavior can only be said to be **rational** with respect to a given normative standard of reference.
- The traditional normative standards of reference are **too strong**.
- Conditional probability logic is the proposed normative standard.
- **Mental probability logic**
  - is proposed as a competence theory of human reasoning
  - combines ideas from nonmonotonic reasoning, logic, and probability theory (coherence, probability intervals)
  - interprets “*if A, then B*” as  $P(B|A)$
  - is about probability propagation from the premises to the conclusion