The new psychology of reasoning: A mental probability logical perspective

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Abstract

Mental probability logic (MPL) has been proposed as a competence theory of human inference. MPL interprets indicative conditionals as conditional events. While recent probabilistic approaches assume an uncertain relation between the premises and the conclusion, the consequence relation remains deductive in MPL. The underlying rationality framework of MPL is *coherence based probability logic*. I discuss cases in which the psychological predictions of MPL on human reasoning about conditionals diverge from psychological predictions which are based on traditional approaches to probability in the context of zero antecedent probabilities. Specifically, I reconstruct a paradox of the material conditional in purely probabilistic terms. Moreover, I report a new experiment on a generalized version of the probabilistic truth table task, which investigates the interpretation of conditionals under incomplete probabilistic knowledge. The data show that—during the course of the experiment—most people shift their interpretation and converge on the conditional event interpretation of conditionals. Finally, I illustrate how the data of this experiment and of experiments on a paradox of the material conditional confirm the psychological predictions of MPL.

Keywords: Mental probability logic, coherence, rationality, conditionals, paradoxes, probabilistic truth table task, zero probability

Introduction

The new paradigm psychology of reasoning is characterized by using probability theory as a rationality framework for human reasoning. In contrast, classical bivalent logic dominated the traditional psychology of reasoning (Evans, 2012; Evans & Over, 2012). In this paper I advocate *Mental probability logic* (MPL), which emerged within the new paradigm psychology of reasoning (e.g., Pfeifer, 2006, 2012, in press; Pfeifer & Kleiter, 2005b, 2009, 2010). Among the various approaches to probability (Hájek, 2011), MPL selects *coherence based probability logic* as a rationality framework for human reasoning. MPL and the coherence approach are "Bayesian" in the sense that probabilities are subjective and conceived as *degrees of belief*. Degrees of belief are psychologically appealing as they are not only formally well-defined in probabilistic terms, but they also elicit psychologically plausible connotations: What people know is evaluated in terms of degrees of belief, since everyday life reasoning is usually based on uncertain and incomplete knowledge. Although people usually do not explicitly mention degrees of belief, they are implicitly attached to sentences in everyday life discourse. Contrary to other Bayesian approaches in the new psychology of reasoning (e.g., Oaksford & Chater, 2007; Harris & Hahn, 2009), MPL does not use Bayes' theorem as the key ingredient for constructing the rationality models. Rather than assuming an *uncertain* relation between the premises and the conclusion (often formalized by variations on Bayes' theorem), MPL investigates the coherent transmission of the uncertainty of the premises to the conclusion. Probabilities are attached to the premises and the relation between the premises and the conclusion remains *deductive*. Consider the probabilistic modus ponens as an example:

$$\begin{array}{ll} A & P(A) = x_1 \\ \text{If } A\text{, then } C & P(C|A) = x_2 \\ \hline C & x_1x_2 \leq P(C) \leq x_1x_2 + 1 - x_1 \end{array}$$

The premise probabilities x_1 and x_2 are transmitted deductively to the conclusion $P(C)$. The two premise probabilities provide enough input to allow for deducing informative coherent probability bounds on the conclusion. For deducing a point value, more information is needed. For example, if $P(C|\neg A) = x_3$ is given as an additional premise, then the conclusion probability is a point value, calculated by $P(C) = x_1x_2 + (1 - x_1)x_3$. Thus, the modus ponens is an example for a situation of *incomplete probabilistic knowledge*, which stresses the importance of *imprecise* (or interval-valued) probabilities. The probability propagation from the premises to the conclusion is deductive, since its justification is provable within coherence based probability theory.

Recently, Oaksford and Chater explained that their approach can be conceived "as enriching conventional logic to give an *inductive* logic—a system of logic that extends deduction to less-than-certain inferences" (2009, p. 107). Their reference to inductive logic and their emphasis on probabilistic consistency preservation (Chater & Oaksford, 2009) is akin to MPL. Yet, there are crucial differences. While inductive logics traditionally define some degree of support of the conclusion by the premises (Hawthorne, 2012), MPL assumes a deductive consequence relation. Instead of using a notion of probabilistic con-

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sistency, which is usually based on the traditional axioms of probability theory, MPL uses the *coherence approach* to probability. The coherence approach to probability goes back to de Finetti (1937/1980, 1970/1974) and has been further developed in the last decades (e.g., Walley, 1991; Lad, 1996; Biazzo & Gilio, 2000; Coletti & Scozzafava, 2002; Galavotti, 2008). In traditional approaches to probability (e.g. Kolmogorov, 1933/1950), unconditional probabilities are *primitive* and conditional probability ($P(C|A)$) is defined via the fraction of the conjunction ($P(A \wedge C)$) and the antecedent probabilities ($P(A)$) of the corresponding conditional ("If A, then C"):

$$P(C|A) =_{def.} \frac{P(A \wedge C)}{P(A)}, \quad \text{if } P(A) > 0\,.$$

Thus, the case when the antecedent probability is equal to zero is *undefined*, as divisions by zero are undefined. To deal with zero antecedent probabilities, Adams suggested that one should—by default—assume that $P(C|A) = 1$, if $P(A) = 0$ (Adams, 1998, p. 57). In principle, Adams' suggestion can be taken as an interesting empirical hypothesis about how people deal with zero antecedent probabilities. Formally, however, this default assumption is unsatisfactory, as it implies that if $P(A) = 0$, then $P(C|A) = P(\neg C|A) = 1$. This is incoherent as $P(C|A)$ plus $P(\neg C|A)$ should add up to one. I will present empirical data, which suggest that people do not make Adams' default assumption.

In the coherence approach, conditional probability ($P(C|A)$) is *primitive* and not defined via the fraction of the conjunction and the antecedent probabilities. This allows for dealing with zero antecedent probabilities. I will illustrate the importance of zero antecedent probabilities in the contexts of probabilistic truth table tasks and of a paradox of the material conditional below.[1]

An important research topic for the new paradigm psychology of reasoning is how people interpret and reason about conditionals. Ramsey's famous footnote can be interpreted as an early version of the now popular psychological hypothesis that people interpret indicative conditionals as *conditional events*:

> "If two people are arguing "If p will q?" and are both in doubt as to p, they are adding p hypothetically to their stock of knowledge and arguing on that basis about q; [. . .] We can say they are fixing their degrees of belief in q given p. If p turns out false, these degrees of belief are rendered *void*" (Ramsey, 1929/1994, p. 155, footnote).

The procedure described in this footnote is called the "Ramsey test" and corresponds to the de Finetti table (de Finetti, 1936/1995, 1970/1974) of the conditional event (see Table 1). This footnote has been reproduced numerous times in the literature, but the last sentence

[1] In this paper I consider synchronic but not diachronic probabilistic assessments (probabilistic updating). Updating extreme probabilities (zero and one) are problematic for traditional approaches. In the coherence approach, however, probability one *can be updated* (Coletti & Scozzafava, 2002, see, e.g., Section 11.6). Moreover, both $P(\text{hypothesis}) = 0$ and $P(\text{hypothesis} \mid \text{evidence}) > 0$ are simultaneously coherent (but not expressible in traditional approaches to probability; see Pfeifer (in press)). Thus, the coherence approach can also deal with the zero-prior problem, which is well-known in the philosophy of induction. For more on updating probabilities in psychology see, e.g., Baratgin and Politzer (2010); Oaksford and Chater (in press); Walliser and Zwirn (2002).

Table 1
Truth conditions of the conditional event ($C|A$), the conjunction ($A \wedge C$) and the material conditional ($A \supset C$) interpretation of an indicative conditional "If A, then C".

State of the world		Interpretation		
A	C	$C\|A$	$A \wedge C$	$A \supset C$
true	true	true	true	true
true	false	false	false	false
false	true	void	false	true
false	false	void	false	true

Table 2
Probability logical form of probabilistic truth table tasks. The premise probabilities (x_1–x_4) uniquely determine the probabilities of various interpretations of the conclusion probability (P(If A, then C)).

$P(A \wedge C)$	=	x_1
$P(A \wedge \neg C)$	=	x_2
$P(\neg A \wedge C)$	=	x_3
$P(\neg A \wedge \neg C)$	=	x_4
$P(\text{If } A, \text{then } C)$	=	?

of this quote is almost always omitted. However, this sentence is crucial for explaining the semantics of the conditional event: *If A, then C* is *void*, if A is false. This means that the truth value of the conditional is undetermined if the corresponding antecedent is false. In contrast, the material conditional $A \supset C$ is true and the conjunction $A \wedge C$ is false, if A is false (see Table 1). The conditional event is not expressible in classical bivalent logic.

MPL interprets indicative conditionals as conditional events. Consequently, the degree of belief in an indicative conditional is evaluated by the corresponding conditional probability. $P(C|A)$ is directly assigned to $C|A$ without presupposing knowledge about $P(A \wedge C)$ and $P(A)$. Of course, $P(A \wedge C)$ and $P(A)$ are used, if they are available to the reasoner.

In the traditional psychology of reasoning, experimental paradigms like Wason's card selection task, truth table tasks, or suppression tasks were developed to investigate how people interpret indicative conditionals. Responses which were consistent with the conditional event semantics were interpreted to be irrational, as they violate the semantics of the material conditional. As a result, conditional event response patterns were dubbed as "defective truth table" patterns. In the new paradigm psychology of reasoning, however, this response pattern was justified by the insight that there is nothing rationally "defective" about this pattern, as it corresponds to the semantics of the conditional event (Baratgin, Over, & Politzer, in press; Pfeifer & Kleiter, 2009, 2011, 2005a, 2010; Politzer, Over, & Baratgin, 2010).

Probabilistic truth table task

One of the most important reasoning tasks of the new psychology of reasoning is the probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003). In this task, the probabilities of all truth table cases are presented as premises and the task consists of inferring the probability of "*If A, then C*". Table 2 shows how the task is formalized in terms of a deductive argument.

The probabilistic truth table task requires first to fix the interpretation of "*If A, then C*" and second to infer the probability. This task allows for inferring the participants' interpretation of the conditional from the responded probability assessments. Contrary to the probabilistic modus ponens, all constituent probabilities (i.e., the probabilities of each line in the truth table) are given. Thus, the task reflects a situation of *full probabilistic knowledge*. Therefore, the psychologically most prominent interpretations of indicative conditionals—conditional event, conjunction, and material conditional—are uniquely determined by the premises: $P(C|A) = x_1/(x_1 + x_2)$, $P(A \wedge C) = x_1$, and $P(A \supset C) = x_1 + x_3 + x_4$, respectively.

Most studies on the probabilistic truth table task paradigm report that just over 50% of the responses are consistent with the conditional event pattern. A non-negligible percentage of the remaining responses match the conjunction interpretation and almost no participant responded by the material conditional interpretation. This speaks against the psychological hypothesis, that the core meaning of basic conditionals[2] corresponds to the material conditional, which was advocated by the theory of mental models (Johnson-Laird & Byrne, 2002).

Conjunction responses could be interpreted as half-way conditional event responses: people correctly form the numerator of the fraction $x_1/(x_1 + x_2)$ but neglect to divide it by the denominator. Matching effects could also explain the conjunction responses. Since the premises are usually given in terms of conjunctions, the conjunction responses may be produced by matching the first premise. Finally, the relatively high percentage of conjunction responses could be caused by the following linguistic ambiguity: many instructions prompt the participant to evaluate the *truth* of a conditional (e.g., Oberauer & Wilhelm, 2003; Evans, Handley, Neilens, & Over, 2007). Both, the conditional event $C|A$ and the conjunction $A \wedge C$ *are true*, if A and C are true. However, both are strictly speaking *not true* if $\neg A$ (though $C|A$ *may be true* if $\neg A$). Thus, if the participants read the instruction such that the experimenter wants to know in which cases the conditional is strictly speaking *true*, then the task cannot differentiate between the conjunction and the conditional event interpretation. To avoid this problem of ambiguity, I advocate to prompt the participant to evaluate to what degree a conditional "*holds*" instead of to what degree a conditional "*is true*".

In a recent study, participants were presented a series of six-sided dice (Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011a). A geometric form in a single color was printed on each side. Form and color were varied systematically, such that each die implied different probabilities that matched the respective interpretations of conditionals. Each task displayed all six sides of a die. The task consisted in assessing whether conditionals (e.g., "If the side shows a square, then the side shows red") *hold* of certain dice. A key differ-

[2]"Basic conditionals" are conditionals that are not evaluable by background knowledge.

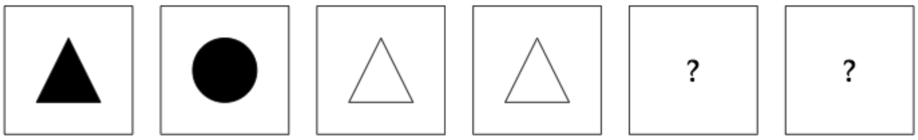
ence between this task and probabilistic truth table tasks in the literature is that in this task the participant can choose how to represent the displayed probabilistic information about forms and colors, without being primed towards a specific logical interpretation of the premise set. The task was presented several times. This revealed an important novel finding: the percentage of conditional event responses shifted from around 40% at the beginning to nearly 80% at the end of the task, with most participants shifting from a conjunction to a conditional event interpretation (Fugard et al., 2011a). This indicates two important points: Firstly, the modal interpretation is the conditional event interpretation, as predicted by MPL. Secondly, a considerable percentage of participants shift their interpretation.

Since full probabilistic knowledge is usually not available in everyday life reasoning, one can doubt the ecological validity of probabilistic truth table tasks. The next section reports a new experiment on a generalized version of the probabilistic truth table task. I investigated how people interpret indicative conditionals under *incomplete* probabilistic knowledge.

Probabilistic truth table task under incomplete knowledge

The new version of the dice task was presented to the participants as follows:

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*).
Question marks indicate covered sides.



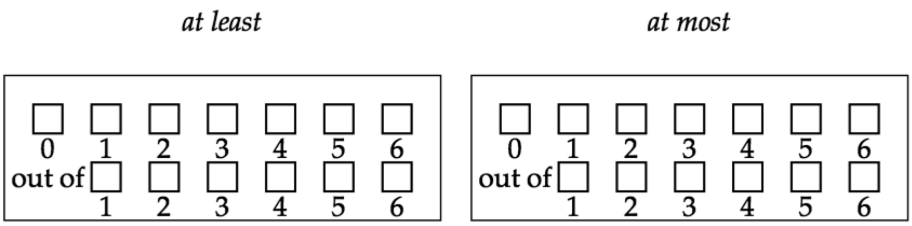
Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *black*, **then** the side shows a *circle*.

The covered sides put the participant into a situation of *incomplete probabilistic knowledge*. Consequently, the competence answers for the three most prominent interpretations of indicative conditionals are *imprecise*. Thus, the response format needs to account for lower and upper bounds, which was formulated as follows:

Answer:



(please tick the appropriate boxes)

In the above example the conditional event response corresponds to “at least 1 out of 4” and “at most 3 out of 4”. The lower and upper bounds for the conjunction interpretation are “1 out of 6” and “3 out of 6”, respectively. The material conditional response corresponds to “at least 3 out of 6” and “at most 5 out of 6”. To reduce priming participants with interval valued responses, the introduction made clear how to respond point values in this response format. Moreover, in some of the tasks all sides were visible, which normatively required to respond by point values.

Twenty (10 female, 10 male) Cambridge University students participated in this experiment. The age ranged between 18 and 27 years ($M = 21.65$, $SD = 2.48$). No students of mathematics, philosophy, computer science, or psychology were included in the sample. Each participant received £ 5 for participation. positiv beschreiben

After introducing the task material and the response format, three tasks were given as warm-up trials. After the warm-up trials, 17 target tasks were presented. For each die task the configuration of color and shape of the depicted figures and the properties mentioned in the conditional were varied. All tasks differ with respect to the three predictions $P(C|A)$, $P(A \wedge C)$, and $P(A \supset C)$. The repetitive presentation of the tasks was necessary to study possible shifts of interpretation. Only the target tasks are included in the data analysis.

Results and discussion

The median lower and upper bound responses in each task are consistent with the conditional event interpretation of indicative conditionals, as predicted by MPL. Overall, there are 340 interval-responses, among which 65.6% are consistent with the conditional event, 5.6% with the conjunction, and only 0.3% are consistent with the material conditional interpretation.

38.3% of the 60 interval responses in the first three target tasks are consistent with the conditional event interpretation. This percentage increased to 83.3% in the last three tasks. This replicates the shifts of interpretation we observed in previous studies (Fugard et al., 2011a; Fugard, Pfeifer, & Mayerhofer, 2011b). During the course of the experiment, people converge towards the conditional event interpretation, which is the competence response predicted by MPL. Figure 1 shows the increase of conditional event responses during the course of the experiment. The frequency of conditional event responses and the item position are strongly correlated ($r(15) = 0.71$, $p < 0.005$).

The data analysis revealed an interesting novel reasoning strategy: for evaluating the lower bound on the conditional, some participants ignored the covered sides of the

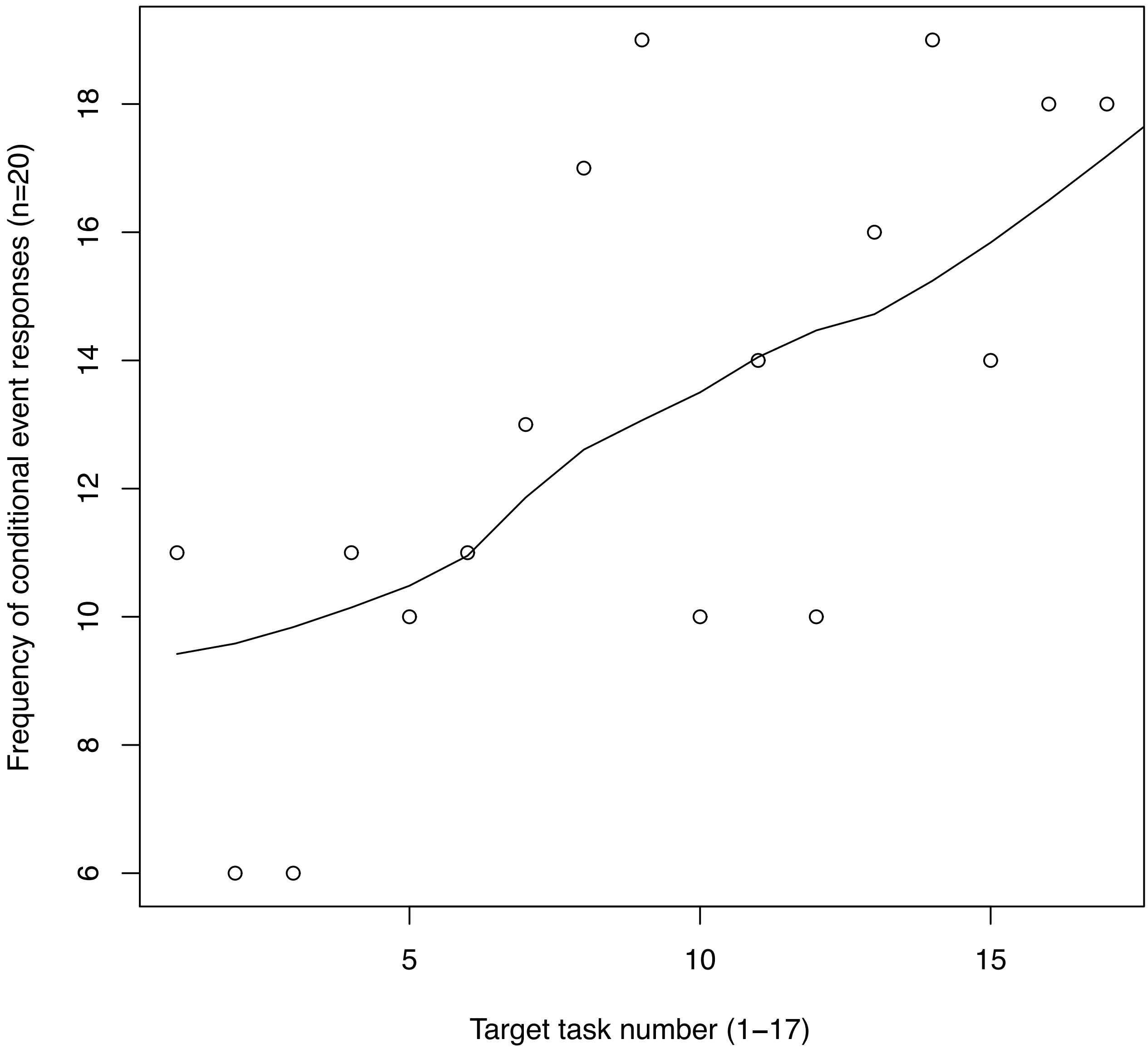


Figure 1. Increase of conditional event responses ($n = 20$). The solid line was generated using the locally weighted scatter plot smoother method (`lowess`, see Cleveland, 1981; implemented in `R`).

respective die and produced conditional event responses based on the visible sides. The upper bound response coincides with the upper bound predicted by the conditional event interpretation. This reasoning strategy would predict the response "at least 1 out of 2" and "at most 3 out of 4" for the above illustrated die example. 11 of the 17 target tasks differentiate between this kind of "half-way conditional event strategy" (HWCE) and the three other strategies. 18.6% of the 220 interval responses in these tasks are consistent with the HWCE strategy. Ignoring the covered sides unburdens the working memory load, which is a possible explanation of the HWCE responses.

Traditional approaches to probability and the coherence approach (which is advocated by MPL) do not differ in their predictions in those probabilistic truth table tasks which are reported in the literature. However, in the last task of the current experiment a reasoning strategy is possible which cannot be formulated within traditional approaches to probability, but which is well defined in the coherence approach: it includes the case in which the antecedent may have probability zero. In this task, each of three sides showed a white circle and three sides were covered. The participants were instructed to assess how sure one can be that the following conditional holds:

If the side facing up shows a *square* (A), **then** the side shows *black* (C).

The material conditional interpretation predicts $0.5 \leq P(A \supset C) \leq 1$ and the conjunction interpretation predicts $0 \leq P(A \wedge C) \leq .5$. There are at least two strategies to solve this task under conditional probability interpretations.

The first strategy presupposes that the participants assume the extreme case (#1) that none of the three sides behind the covers shows a square, which can be represented by $P(A) = 0$. What is the probability of "If A, then C" in case #1? As explained above, in this case $P(C|A)$ is undefined in traditional approaches to probability: the participants should opt out of the task, e.g., by reporting a feeling of oddness to the experimenter. Alternatively, the participants should respond by $P(C|A) = 1$, if they make Adams' (1998, p. 57) default assumption. In the coherence approach, however, the conditional event is well-defined and the unit interval is predicted, $0 \leq P(C|A) \leq 1$.

In a follow-up study involving similar tasks, participants were prompted to explain their solutions. It turned out that none of the participants considered case #1 for solving this task. Instead, most participants reported that they considered the two cases where only black squares (extreme case #2) or only white squares (extreme case #3) are hidden behind the covers. In the present experiment, participants were not interviewed. However, if the second strategy was dominant in this experiment as well, which is highly probable, the predictions of MPL, traditional approaches to probability and Adams' system coincide: if the participants interpret the indicative conditional as a conditional probability, then the participants respond by the unit interval. 18 out of 20 participants responded by the unit interval which empirically endorses the conditional probability interpretation, as predicted by MPL.

In the next section, I discuss a case, where traditional approaches to probability and the coherence approach diverge with respect to the predictions in the case of zero antecedent probabilities.

A paradox of the material conditional

Paradoxes of the material conditional arise if natural language conditionals are interpreted by material conditionals. As an example, consider the following argument form:

$$\frac{C}{\text{If } A \text{, then } C}$$

If the conclusion is interpreted as a material conditional, then the following natural language instance of this argument is logically valid:

$$\frac{\text{Bill Gates is a billionaire.}}{\text{If Bill Gates is bankrupt, Bill Gates is a billionaire.}}$$

This is paradoxical since the believability of the premise is high and the believability of the conclusion is low, while the argument is logically valid under the material conditional interpretation.

In general, a psychologically appropriate rationality framework should predict that the inference from "C" to "If A, then C" is non-informative. This intuition is satisfied if the conditional is interpreted as a conditional event.

From probabilistic points of view, the above argument form reads:

$\frac{P(C) = x}{x \leq P(A \supset C) \leq 1}$	$\frac{P(C) = x, \text{ if } x < 1}{0 \leq P(C\vert A) \leq 1}$	$\frac{P(C) = x}{0 \leq P(C\vert A) \leq 1}$
(material conditional)	(traditional approaches)	(coherence approach)

Bonnefon and Politzer (2011, p. 154) point out that in the special case where the premise is certain (i.e., reaches probability one), the conclusion is certain in the conditional event interpretation. This is true under the assumption that $P(A) > 0$: if $P(C) = 1$, then $P(A \wedge C) = P(A)$. Consequently—using the traditional definition of conditional probability—we obtain $P(C|A) = P(A)/P(A)$, which is equal to one, provided $P(A) > 0$. However, $P(C|A)$ is undefined, if $P(A) = 0$. As explained above, in Adams' system $P(C|A)$ is per default equal to one in this case. However, both "undefined" and "probability one" are technically unsatisfactory and psychologically counterintuitive.

In the coherence approach, $0 \leq P(C|A) \leq 1$ is coherent for *all* probability assessments of the premise, even if $P(C) = 1$ (for a detailed proof see Pfeifer (in press)). Whereas traditional approaches to probability need additional assumptions (e.g., pragmatic considerations) to explain the non-informativeness of the argument form, the coherence approach can explain it in purely semantic terms. I do not argue against the use of pragmatic principles in general, but for this paradox of the material conditional, MPL works in purely semantic terms and pragmatic principles appear redundant.

Two experiments investigated this argument form with basic conditionals (i.e., conditionals that are not evaluable by background knowledge). The clear majority of participants understood that the argument form is non-informative (Pfeifer & Kleiter, 2011). In these experiments the premise probabilities were formulated in terms of percentages and verbal descriptions of the degrees of belief. In one of the tasks the degree of belief in the premise was formulated by "absolutely certain", which can be read as $P(C) = 1$. As predicted by MPL, 69% of the participants evaluated the conditional as non-informative

(Pfeifer & Kleiter, 2011, Experiment 1, $n = 16$). This is strong evidence in favor of the coherence approach and evidence against the material conditional interpretation of conditionals, the traditional definition of conditional probability and Adams' default assumption.

In the above instantiation with "Bill Gates", we know that being bankrupt is incompatible with being a billionaire. This is formalized in coherence based probability logic as follows:

Bill Gates is a billionaire.	$P(C) = 1$
Being bankrupt logically excludes being a billionaire.	$A \wedge C \equiv \perp$
If Bill Gates is bankrupt, Bill Gates is a billionaire.	$P(C\|A) = 0$

"$\perp$" denotes logical falsum. This formalization captures the intuition that the believability in each premise is high and the believability of the conclusion in purely semantic terms, without referring to pragmatic principles.

An alternative explanation of the paradox, which is based on pragmatics, has been proposed by Bonnefon and Politzer (2011). The authors suggest the following two pragmatic conditions to explain why some instances of the paradox appear to be intuitively acceptable inferences, if these conditions are met:

> "(p1) the truth of [the antecedent] x has bearings on the relevance of asserting [the consequent] y; and (p2) the speaker can reasonably be expected not to be in a position to assume that x is false" (p. 146).

At the time of writing this paper, it seems clear that (p2) is violated in the Bill Gates example: we can be quite sure that the antecedent (Bill Gates is bankrupt) is false. Moreover, because the antecedent logically excludes the consequent, (p1) is violated. Thus, the Bill Gates inference is not intuitively acceptable according to this pragmatic account.

For an intuitively acceptable instance of the paradox consider the following example:

Katrin is in the library.
—————————————————
If Katrin is not in the department, then she is in the library.

Here, (p1) and (p2) are satisfied.

In the framework of MPL, (p2) can be interpreted as "the probability of the antecedent is high" (where "high" means, e.g., greater than .5). Adding $P(\neg A)$ to the premise set,[3] we obtain (in general) the following informative argument form:[4]

$$\frac{\begin{array}{c} P(C) = x \\ P(\neg A) = y \end{array}}{\max\left\{0, \frac{x+y-1}{y}\right\} \leq P(C|\neg A) \leq \min\left\{\frac{x}{y}, 1\right\}}$$

Thus, adding a probabilistic version of (p2) to the premise set is sufficient to obtain a probabilistically informative argument form. However, probabilistic informativeness does neither guarantee a high conclusion probability nor the acceptability of the conclusion. Thus, for the acceptability of the conclusion, more assumptions are needed.

[3]Using the Katrin example, "$\neg A$" denotes "Katrin is not in the department" and "C" denotes "Katrin is in the library".

[4]This argument form is a special case of the "cautious monotonicity" rule of the basic nonmonotonic reasoning System P, for which Gilio (2002) elaborated a coherence based probability semantics.

Another interesting possibility to obtain a probabilistically informative version of the paradox is to interpret some instances of the paradox as or–to–if inferences. For pragmatic reasons, people may sometimes interpret the premise C by the disjunction $A \vee C$: Katrin is either in the department or in the library. Then, as coherence requires $0 \leq P(C|\neg A) \leq P(A \vee C)$, the conditional may obtain a positive probability. If the disjunction is justified non-constructively (i.e, Katrin is either in the department or in the library, but it is not clear where she is), the inference from "A or C" to "If not-A, then C" appears to be strong, where "strong" means here that the conditional probability is "close" to the disjunction probability (Gilio & Over, 2012; Over, Evans, & Elqayam, 2010).

This explanation, the pragmatic account by Bonnefon and Politzer (2011), and the explanation based on the probabilistic version of (p2) call for future empirical work. In general, enriching the premise sets of well-known argument forms by logical and/or probabilistic constraints generates fruitful new empirical hypotheses for the new paradigm psychology of reasoning.

Concluding remarks

The new paradigm psychology of reasoning uses probability theory as a rationality framework for human inference. In this paper I advocated *Mental probability logic* (MPL), which uses the coherence approach to probability as a rationality framework. While I endorse the paradigm shift from logic to probability, I think it is high time to look more closely on the specific interpretations of probability that are used in the new paradigm psychology of reasoning. Specifically, I illustrated that there are important differences between the psychological predictions derived from the various approaches to probability. I explained why MPL makes normatively and descriptively adequate predictions on reasoning about conditionals which involve zero antecedent probabilities.

Zero antecedent probabilities may also be relevant in the context of counterfactual conditionals ("If A were the case, C would be the case"). A conditional is *counterfactual*, if its antecedent is (known to be) factually false. If the antecedent is factually false, then the conditional event $C|A$ is void. One way of representing knowledge about factual falsehood of A is to assign a zero probability to A. If $P(A) = 0$, then the corresponding conditional probability $P(C|A)$ may obtain any value from the unit interval $[0, 1]$. Gilio and Over (2012, p. 121) argue that $P(C|A)$ may then reflect the probability of the corresponding counterfactual conditional. Whether one should assign probabilities to known factual truths or falsehoods is a deep philosophical problem which needs to be addressed elsewhere.

Traditional probability theory has already been discussed in the psychology of judgment and decision making (JDM) for some decades now (e.g., Gilovich, Griffin, & Kahneman, 2002), but the reasoning and JDM communities were traditionally more or less separated. One reason is that both pursue different research goals: psychologists of reasoning aim to investigate reasoning processes, whereas the primary goal of the JDM community is to analyze human judgments and decisions. As JDM research and the new reasoning paradigm share the use of probability for psychological model building, this separation could be surmounted in order to develop joint research agendas. The new paradigm psychology of reasoning supports a holistic perspective on reasoning and JDM: both psychological sub-disciplines should be integrated. I propose that reasoning is prior to judgment and decision making: the conclusions obtained from the premises by acts of reasoning can constitute a basis for justifying judgments and decisions.

In MPL degrees of belief are attached to the premises and transmitted deductively to the conclusion. Framing human inference tasks as deductive reasoning tasks allows for a formally well-defined approach for investigating human inference. Approaches that impose uncertain relations between the premises and the conclusions are unable to map appropriately the probability-logical structure of the argument, if conditional events are included in the premise set. It is far from clear, for example, how to formalize an uncertain relation between the premises and the conclusion of the modus ponens: $P(C \mid \{A \oplus (C|A)\})$ is not well-defined (where "$\oplus$" denotes some combination of the premises).[5] However, if the probabilistic modus ponens is formalized as a *deductive* argument, technical problems of conditionalizing on conditionals are avoided, the logical structure is mapped and the argument is formalized in well-defined terms (see the probabilistic modus ponens in the introductory section). Moreover, careful probability-logical task analyses allow for investigating important semantic distinctions, e.g., wide- versus narrow-scope readings of negations, which generate interesting and testable new psychological hypotheses on human inference (see, e.g. Pfeifer, 2012).

Finally, I remarked that the probabilistic truth table task (which is one of the standard tasks of the new paradigm psychology of reasoning) investigates reasoning under *complete* probabilistic knowledge. In everyday life situations, however, full probabilistic knowledge is usually not available. Therefore, the new psychology of reasoning should focus on investigating reasoning under *incomplete* probabilistic knowledge to enhance the ecological validity of the inference tasks. This typically involves interval-valued probabilities. Coherence based probability logic provides the necessary methodological tools for handling interval-valued probabilities.

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[5]See Dietz and Douven (2010) and Gilio and Sanfilippo (2013a, 2013b) for formal work on nested and iterated conditionals.

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