Probability logic, language, and the mind

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Three levels of description (Marr, 1982)

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(photo taken by N. Pfeifer at Black Magic Bar in Rīga)

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- Computational (problem description/task analysis)
- Algorithmic (representations and processes)
- Hardware

Mental probability logic (Pfeifer, 2006b, 2012a, 2012b, 2013a, 2014; Pfeifer & Kleiter, 2005)

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- **Key idea: Uncertainty is propagated deductively from the premises to the conclusion**
- Rationality framework: coherence based probability logic

Coherence based probability logic

- Coherence
  - de Finetti, and {Coletti, Gilio, Lad, Regazzini, Sanfilippo, Scozzafava, Vantaggi, Walley, ...}
  - degrees of belief
  - complete algebra is not required
  - many probabilistic approaches define $p(B|A)$ by

$$\frac{p(A \wedge B)}{p(A)} \quad \text{and assume that} \quad p(A) > 0$$

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what if $p(A) = 0$?

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  - zero probabilities are exploited to reduce the complexity
  - imprecision

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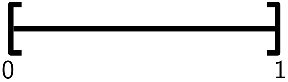
    in the coherence approach, conditional probability, $p(B|A)$, is primitive
  - zero probabilities are exploited to reduce the complexity
  - imprecision
- Probability logic
  - uncertain argument forms
  - deductive consequence relation
  - **propagation of the uncertainties from the premises to the conclusions**

Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = x$
A	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$

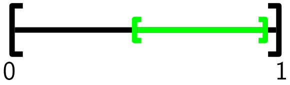
Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C \mid A) = x$
A	$p(A) = y$
C	$xy \leq p(C) \leq xy + 1 - x$



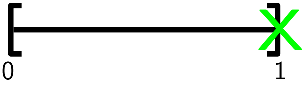
Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = .90$
A	$p(A) = .50$
C	$.45 \leq p(C) \leq .95$



Example: Probabilistic modus ponens

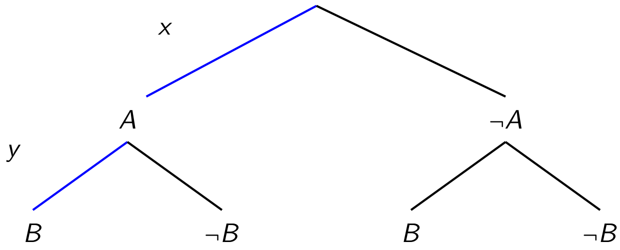
(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = 1$
A	$p(A) = 1$
C	$p(C) = 1$



Example: Probabilistic modus ponens

(Modus Ponens)	(Probabilistic modus ponens)
If A, then C	$p(C\|A) = 0$
A	$p(A) = 0$
C	$0 \leq p(C) \leq 1$

from $\boxed{P(A) = x \text{ and } P(B|A) = y}$ infer $\boxed{P(B)}$



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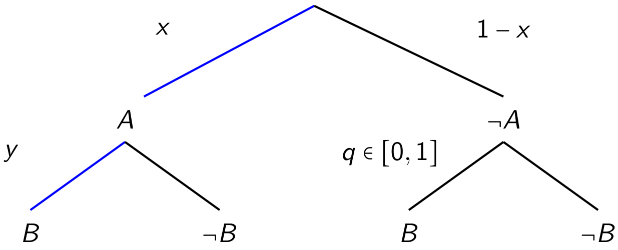

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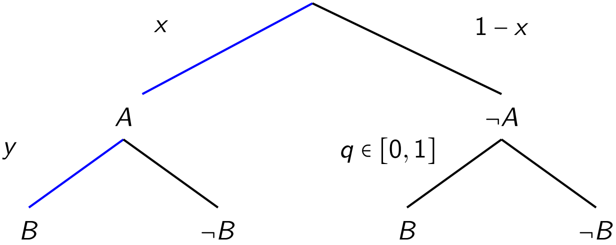
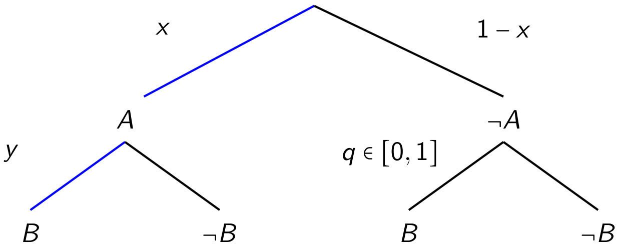


$$P(B) = \underbrace{P(A)}_{x}\,\underbrace{P(B|A)}_{y} + \underbrace{P(\neg A)}_{1-x}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

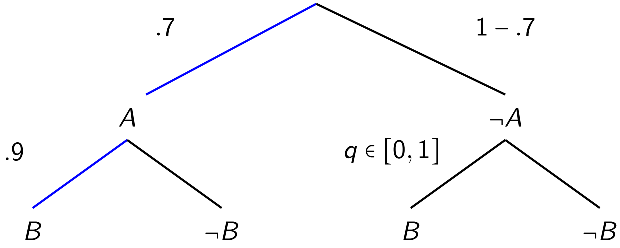
from $\boxed{P(A) = x \text{ and } P(B|A) = y}$ infer $\boxed{P(B)}$



$$P(B) = \underbrace{P(A)}_{x}\,\underbrace{P(B|A)}_{y} + \underbrace{P(\neg A)}_{1-x}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{xy}_{if\ \ q=0} \le P(B) \le \underbrace{xy + (1-x)}_{if\ \ q=1}$$

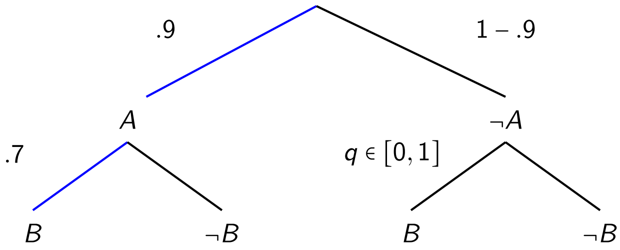
from $\boxed{P(A) = .7 \text{ and } P(B|A) = .9}$ infer $\boxed{P(B)}$



$$P(B) = \underbrace{P(A)}_{.7}\,\underbrace{P(B|A)}_{.9} + \underbrace{P(\neg A)}_{1-.7}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{.63}_{if\ \ q=0} \leq P(B) \leq \underbrace{.93}_{if\ \ q=1}$$

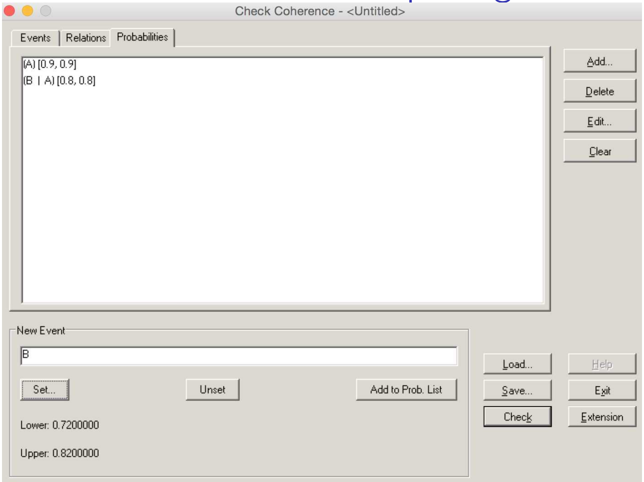
from $\boxed{P(A) = .9 \text{ and } P(B|A) = .7}$ infer $\boxed{P(B)}$



$$P(B) = \underbrace{P(A)}_{.9}\,\underbrace{P(B|A)}_{.7} + \underbrace{P(\neg A)}_{1-.9}\,\underbrace{P(B|\neg A)}_{q\in[0,1]}$$

$$\underbrace{.63}_{if\ \ q=0} \leq P(B) \leq \underbrace{\mathbf{.73}}_{if\ \ q=1}$$

Check Coherence software package



...this software is maintained by Andrea Capotorti and is available here (Baioletti et al., 2016):

http://www.dmi.unipg.it/~upkd/paid/software.html

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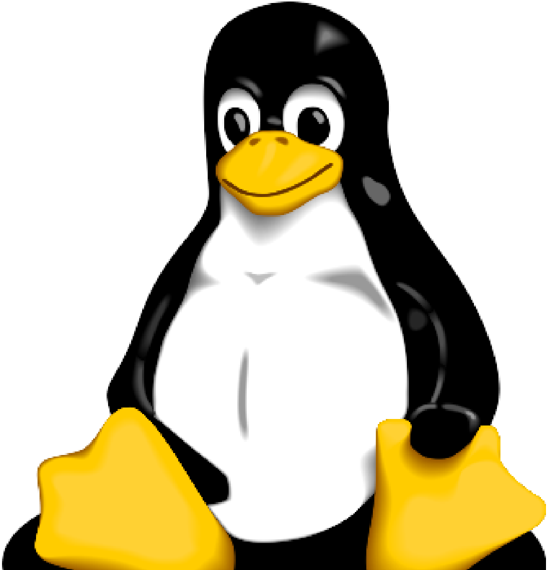
The Tweety problem

The Tweety problem (picture© by L. Ewing, S. Budig, A. Gerwinski; http://commons.wikimedia.org)



The Tweety problem (picture© by ytse19; http://mi9.com/flying-tux_35453.html)

System P: Rationality postulates for nonmonotonic reasoning (Kraus, Lehmann, & Magidor, 1990)

Reflexivity (axiom): $\alpha \mid\!\sim \alpha$

Left logical equivalence:

from $\models \alpha \equiv \beta$ and $\alpha \mid\!\sim \gamma$ infer $\beta \mid\!\sim \gamma$

Right weakening:

from $\models \alpha \supset \beta$ and $\gamma \mid\!\sim \alpha$ infer $\gamma \mid\!\sim \beta$

Or: from $\alpha \mid\!\sim \gamma$ and $\beta \mid\!\sim \gamma$ infer $\alpha \vee \beta \mid\!\sim \gamma$

Cut: from $\alpha \wedge \beta \mid\!\sim \gamma$ and $\alpha \mid\!\sim \beta$ infer $\alpha \mid\!\sim \gamma$

Cautious monotonicity:

from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \wedge \beta \mid\!\sim \gamma$

And (derived rule): from $\alpha \mid\!\sim \beta$ and $\alpha \mid\!\sim \gamma$ infer $\alpha \mid\!\sim \beta \wedge \gamma$

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$\alpha \mid\!\sim \beta$ is read as If α, $\underbrace{\textit{normally}}_{?}$ β

Probabilistic version of System P (Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

Name	*Probability logical version*
Left logical equivalence	$\models(E_1 \equiv E_2), P(E_3\|E_1) = x \therefore P(E_3\|E_2) = x$
Right weakening	$P(E_1\|E_3) = x, \models(E_1 \supset E_2) \therefore P(E_2\|E_3) \in [x, 1]$
Cut	$P(E_2\|E_1 \wedge E_3) = x, P(E_1\|E_3) = y$
	$\therefore P(E_2\|E_3) \in [xy, 1 - y + xy]$
And	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
	$\therefore P(E_2 \wedge E_3\|E_1) \in [\max\{0, x + y - 1\}, \min\{x, y\}]$
Cautious monotonicity	$P(E_2\|E_1) = x, P(E_3\|E_1) = y$
	$\therefore P(E_3\|E_1 \wedge E_2) \in [\max\{0, (x+y-1)/x\}, \min\{y/x, 1\}]$
Or	$P(E_3\|E_1) = x, P(E_3\|E_2) = y$
	$\therefore P(E_3\|E_1 \vee E_2) \in [xy/(x+y-xy), (x+y-2xy)/(1-xy)]$
Transitivity	$P(E_2\|E_1) = x, P(E_3\|E_2) = y \therefore P(E_3\|E_1) \in [0, 1]$
Contraposition	$P(E_2\|E_1) = x \therefore P(\neg E_1\|\neg E_2) \in [0, 1]$
Monotonicity	$P(E_3\|E_1) = x \therefore P(E_3\|E_1 \wedge E_2) \in [0, 1]$

Probabilistic version of System P (Gilio (2002); Table 2 Pfeifer and Kleiter (2009))

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Cut	$P(E_2 \vert E_1 \wedge E_3) = x, P(E_1 \vert E_3) = y$
	$\therefore P(E_2 \vert E_3) \in [xy, 1 - y + xy]$
And	$P(E_2 \vert E_1) = x, P(E_3 \vert E_1) = y$
	$\therefore P(E_2 \wedge E_3 \vert E_1) \in [\max\{0, x + y - 1\}, \min\{x, y\}]$
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Monotonicity	$P(E_3\|E_1) = x \therefore P(E_3\|E_1 \wedge E_2) \in [0, 1]$

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. . . probabilistically non-informative

The Tweety problem (Pfeifer, 2012b)

$\mathfrak{P}1$	$P[\text{Fly}(x)\vert\text{Bird}(x)] = .95.$	*(Birds can normally fly.)*
$\mathfrak{P}2$	$\text{Bird}(\text{Tweety}).$	*(Tweety is a bird.)*
$\mathfrak{C}1$	$P[\text{Fly}(\text{Tweety})] = .95.$	*(Tweety can normally fly.)*

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$\mathfrak{P}3$	$\text{Penguin}(\text{Tweety}).$	*(Tweety is a penguin.)*
$\mathfrak{P}4$	$P[\text{Fly}(x)\vert\text{Penguin}(x)] = .01.$	*(Penguins normally can't fly.)*
$\mathfrak{P}5$	$P[\text{Bird}(x)\vert\text{Penguin}(x)] = .99.$	*(Penguins are normally birds.)*
$\mathfrak{C}2$	$P[\text{Fly}(\text{Tweety}) \mid \text{Bird}(\text{Tweety}) \wedge \text{Penguin}(\text{Tweety})] \in [0, .01].$	

(If Tweety is a bird and a penguin, normally Tweety can't fly.)

The Tweety problem (Pfeifer, 2012b)

$\mathfrak{P}1$	$P[\text{Fly}(x)\|\text{Bird}(x)] = .95.$	*(Birds can normally fly.)*
$\mathfrak{P}2$	Bird(Tweety).	*(Tweety is a bird.)*
$\mathfrak{C}1$	$P[\text{Fly}(\text{Tweety})] = .95.$	*(Tweety can normally fly.)*

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$\mathfrak{C}2$	$P[\text{Fly}(\text{Tweety}) \mid \text{Bird}(\text{Tweety}) \wedge \text{Penguin}(\text{Tweety})] \in [0, .01].$ *(If Tweety is a bird and a penguin, normally Tweety can't fly.)*	

The probabilistic modus ponens justifies $\mathfrak{C}1$ and cautious monotonicity justifies $\mathfrak{C}2$.

Example 1: (Cautious) monotonicity

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{(A \wedge C) \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(B|A \wedge C) \leq 1}$

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- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(B|A \wedge C) \leq 1}$
  But: from $\boxed{P(A \supset B) = x}$ infer $\boxed{x \leq P((A \wedge C) \supset B) \leq 1}$

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- In logic
  from $\boxed{A \supset B}$ infer $\boxed{(A \wedge C) \supset B}$
- In probability logic
  from $\boxed{P(B|A) = x}$ infer $\boxed{0 \leq P(B|A \wedge C) \leq 1}$
  But: from $\boxed{P(A \supset B) = x}$ infer $\boxed{x \leq P((A \wedge C) \supset B) \leq 1}$
- Cautious monotonicity (Gilio, 2002)
  from $\boxed{P(B|A) = x \text{ and } P(C|A) = y}$
  infer $\boxed{\max(0, (x + y - 1)/x) \leq P(C|A \wedge B) \leq \min(y/x, 1)}$

Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

exactly 72% wear a black suit.

Example task: Monotonicity (Pfeifer & Kleiter, 2003)

About the guests at a prom we know the following:

exactly 72% wear a black suit.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

Example task: Cautious monotonicity (Pfeifer & Kleiter, 2003)

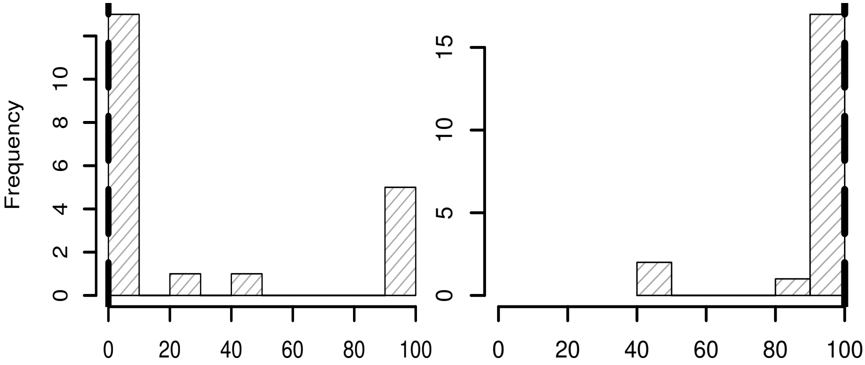
About the guests at a prom we know the following:

exactly 72% wear a black suit.
exactly 63% wear glasses.

Imagine all the persons of this prom who wear glasses.

How many of the persons wear a black suit,
given they are at this prom and wear glasses?

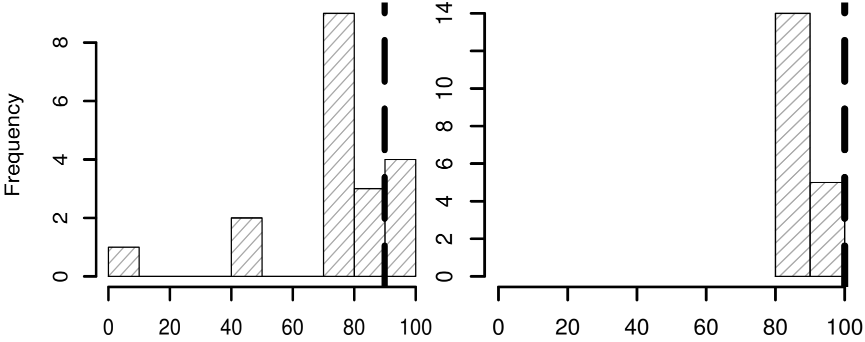
Results – Monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

($n_1 = 20$)

Results – Cautious monotonicity (Example Task 1; Pfeifer and Kleiter (2003))



lower bound responses upper bound responses

$(n_2 = 19)$

Example 2: Contraposition

- In logic
  from $\boxed{A \supset B}$ infer $\boxed{\neg B \supset \neg A}$

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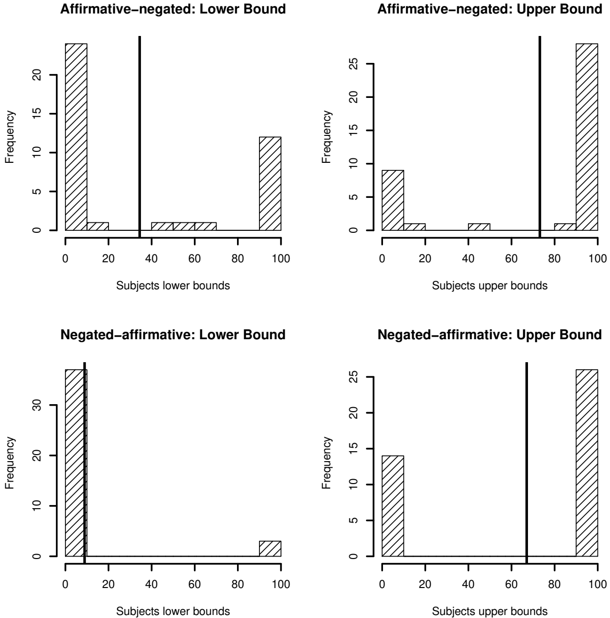
- But

$$P(A \supset B) = P(\neg B \supset \neg A)$$

Results Contraposition (n_1 = 40, n_2 = 40; Pfeifer and Kleiter (2006b))



Modus tollens vs. Contraposition (Pfeifer, 2014, *Studia Logica*)

Consider if A, then B.
Will not-A, if not-B?

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$$P(B|A) = x\,, \quad P(\neg B) = y$$
$$\models 0 \leq \theta \leq P(\neg A) \leq 1$$

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is probabilistically informative
i.e., x and y constrain $P(\neg A)$

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bounds are 0 and 1

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the probabilistic modus tollens
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if $x + y \le 1$, $\theta = \frac{1-x-y}{1-x}$
if $x + y > 1$, $\theta = \frac{x+y-1}{x}$

$\mathfrak{P}1$ If A, then B

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$$P(B|A) = x$$
$$\vDash 0 \le P(\neg A|\neg B) \le 1$$

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Example 3: (Cumulative) transitivity

$$A \to B,\ B \to C, \text{ therefore } A \to C$$

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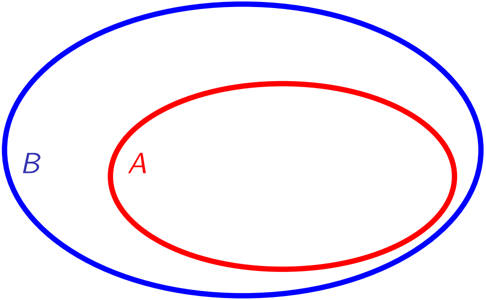
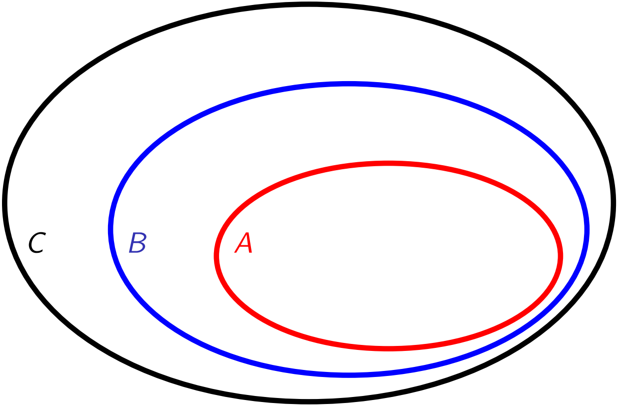


Example 3: (Cumulative) transitivity

$A \rightarrow B$, $B \rightarrow C$, therefore $A \rightarrow C$



Example 3: (Cumulative) transitivity

$$A \mathrel{|\!\sim} B,\ B \mathrel{|\!\sim} C,\ \text{~~therefore~~}\ A \mathrel{|\!\sim} C$$

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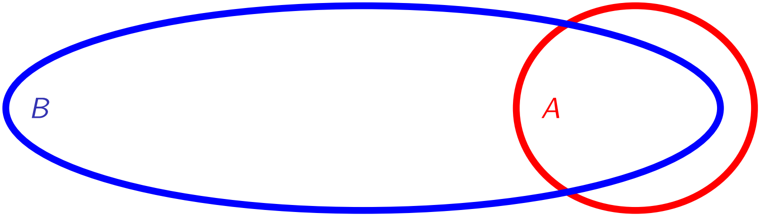
$A \mid\!\sim B$, $B \mid\!\sim C$, ~~therefore~~ $A \mid\!\sim C$



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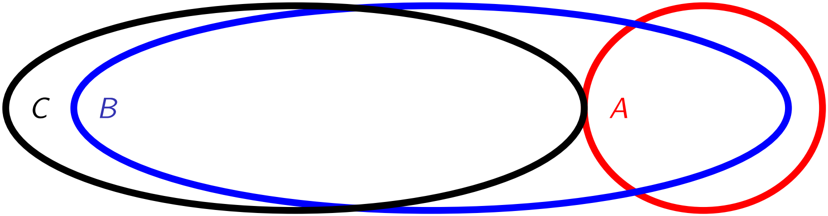

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Example 3: (Cumulative) transitivity

- Transitivity in logic
  from $A \supset B$ and $B \supset C$ infer $A \supset C$
- Transitivity in probability logic
  from $P(B|A) = x$ and $P(C|B) = y$ infer $P(C|A) \in [0, 1]$
- CUT (CUmulative Transitivity)
  from $P(B|A) = x$ and $P(C|A \wedge B) = y$
  infer $P(C|A) \in [xy, 1 - x + xy]$

Modus ponens as a special case of CUT

CUT (Gilio, 2002):

$$\frac{\begin{array}{c} p(B|A) = x \\ p(C|A \wedge B) = y \end{array}}{xy \leq p(C|A) \leq xy + 1 - x}$$

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$$\frac{\begin{array}{c} p(B|A) = x \\ p(C|A \wedge B) = y \end{array}}{xy \leq p(C|A) \leq xy + 1 - x}$$

Let $A \equiv \top$, then

$$\frac{\begin{array}{c} p(B|\top) = x \\ p(C|\top \wedge B) = y \end{array}}{xy \leq p(C|\top) \leq xy + 1 - x}$$

Modus ponens as a special case of CUT

CUT (Gilio, 2002):

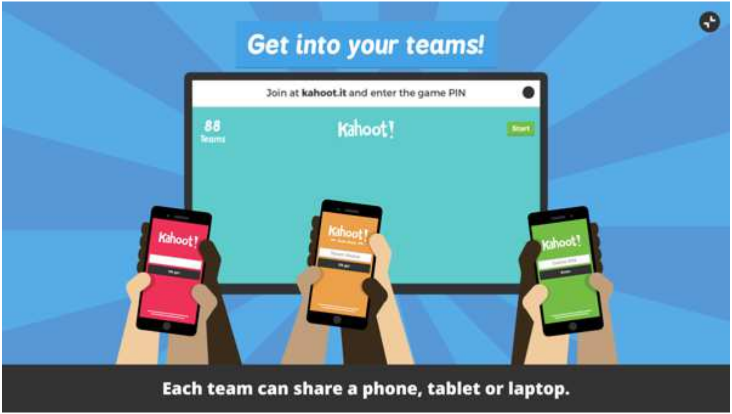
$$\frac{\begin{array}{c} p(B|A) = x \\ p(C|A \wedge B) = y \end{array}}{xy \leq p(C|A) \leq xy + 1 - x}$$

Let $A \equiv \top$. Since $p(E) =_{def} p(E|\top)$ and $p(E \wedge \top) = p(E)$, we obtain:

Modus ponens:

$$\frac{\begin{array}{c} p(B\quad) = x \\ p(C|\quad B) = y \end{array}}{xy \leq p(C\quad) \leq xy + 1 - x}$$

Time for a quiz!



...and go to

`kahoot.it`

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Problematic conditional introduction inferences

Paradoxes of the material conditional, e.g.,

$$\frac{B}{\text{If } A\text{, then } B}\ \text{(Paradox 1)} \qquad \frac{\text{Not: } A}{\text{If } A\text{, then } B}\ \text{(Paradox 2)}$$

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This matches the data (Pfeifer & Kleiter, 2011).

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Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:

If $P(B) = 1$, then $P(A \wedge B) = P(A)$.

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Paradox 1: Special case covered in the coherence approach, but not covered in the standard approach to probability:
If $P(B) = 1$, then $P(A \wedge B) = P(A)$. Thus, $P(B|A) = \frac{P(A \wedge B)}{P(A)} = \frac{P(A)}{P(A)} = 1$, if $P(A) > 0$.

Inf. versions of the paradoxes (Pfeifer, 2014)

From $\Pr(B) = 1$ and $A \wedge B \equiv \bot$ infer $\Pr(B \mid A) = 0$ is coherent.

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$$\max\left\{0, \frac{x+y-1}{y}\right\} \leqslant \Pr(B \mid A) \leqslant \min\left\{\frac{x}{y}, 1\right\} \text{ is coherent.}$$

. . . a special case of the cautious monotonicity rule of System P (Gilio, 2002).

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Probabilistic truth table task (Evans, Handley, & Over, 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
P(A \wedge C) & = & x_1 \\
P(A \wedge \neg C) & = & x_2 \\
P(\neg A \wedge C) & = & x_3 \\
P(\neg A \wedge \neg C) & = & x_4 \\
\hline
P(\text{If } A\text{, then } C) & = & ?
\end{array}
$$

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$$\begin{array}{rcl} P(A \wedge C) & = & x_1 \\ P(A \wedge \neg C) & = & x_2 \\ P(\neg A \wedge C) & = & x_3 \\ P(\neg A \wedge \neg C) & = & x_4 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Conclusion candidates:

- $P(A \wedge C) = x_1$
- $P(C|A) = x_1/(x_1 + x_2)$
- $P(A \supset C) = x_1 + x_3 + x_4$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$
\begin{array}{rcl}
P(A \wedge C) & = & x_1 = .25 \\
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\hline
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\end{array}
$$

Conclusion candidates:

- $P(A \wedge C) = x_1 = .25$
- $P(C|A) = x_1/(x_1 + x_2) = .50$
- $P(A \supset C) = x_1 + x_3 + x_4 = .75$

Probabilistic truth table task (Evans et al., 2003; Oberauer & Wilhelm, 2003)

$$\begin{array}{rcl} P(A \wedge C) & = & x_1 \\ P(A \wedge \neg C) & = & x_2 \\ P(\neg A \wedge C) & = & x_3 \\ P(\neg A \wedge \neg C) & = & x_4 \\ \hline P(\text{If } A, \text{ then } C) & = & ? \end{array}$$

Main results:

- More than half of the responses are consistent with $P(C|A)$
- Many responses are consistent with $P(A \wedge C)$

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(Fugard, Pfeifer, Mayerhofer, & Kleiter, 2011, *Journal of Experimental Psychology: LMC*)

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Key feature:

- Reasoning under complete probabilistic knowledge

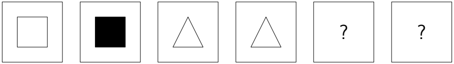
Experiment

Motivation

- probabilistic truth table task with incomplete probabilistic knowledge
- Is the conditional event interpretation still dominant?
- Are there shifts of interpretation?

Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



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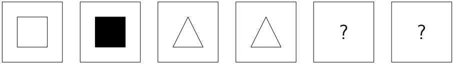

Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

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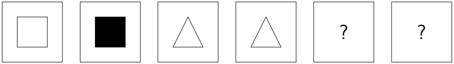

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Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

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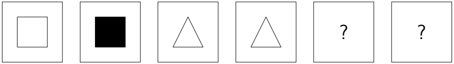

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> **If** the side facing up shows *white*, **then** the side shows a *square*.

Answer:

at least

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6
out of ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

at most

☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6
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(please tick the appropriate boxes)

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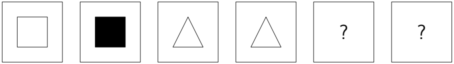

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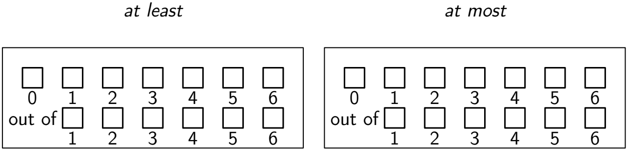
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Cond. event: *at least* 1 out of 5 and *at most* 3 out of 5



(please tick the appropriate boxes)

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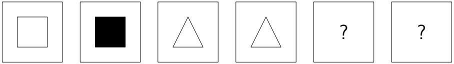

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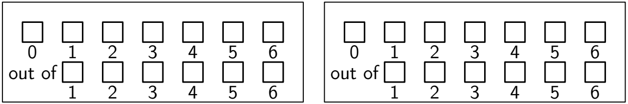
Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Conjunction: *at least* 1 out of 6 and *at most* 3 out of 6

at least *at most*



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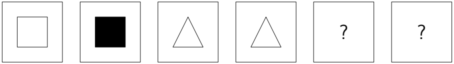
Example: Task 5 (Pfeifer, 2013a, *Thinking & Reasoning*)

Illustrated here are all sides of a six-sided die. The sides have two properties: a color (*black* or *white*) and a shape (*circle*, *triangle*, or *square*). Question marks indicate covered sides.



Imagine that this die is placed in a cup. Then the cup is randomly shaken. Finally, the cup is placed on the table so that you cannot see what side of the die shows up.

Question: How sure can you be that the following sentence holds?

If the side facing up shows *white*, **then** the side shows a *square*.

Answer: Mat. cond.: *at least* 2 out of 6 and *at most* 4 out of 6

at least							
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

at most							
	☐ 0	☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6
out of		☐ 1	☐ 2	☐ 3	☐ 4	☐ 5	☐ 6

(please tick the appropriate boxes)

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Sample

- 20 Cambridge University students
- 10 female, 10 male
- between 18 and 27 years old (mean: 21.65)
- no students of mathematics, philosophy, computer science, or psychology

Experiment (Pfeifer, 2013a, *Thinking & Reasoning*)

Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional

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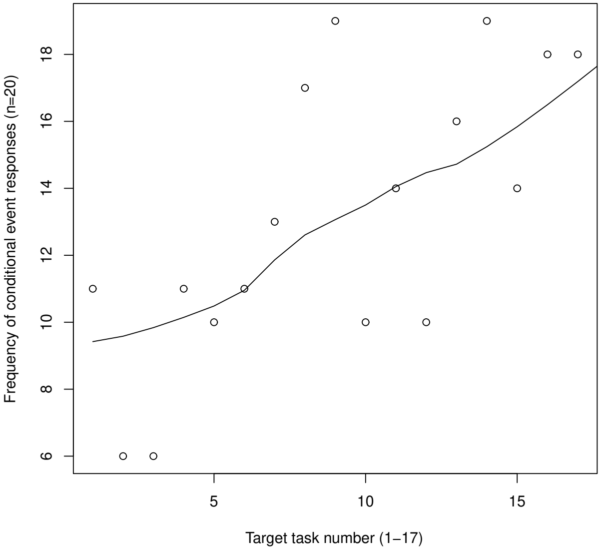
Set-up

- 20 tasks, three "warming-up tasks"
- all tasks differentiate between material conditional, conjunction, and conditional event interpretation

Results

- Overall (340 interval responses)
  - 65.6% consistent with conditional event
  - 5.6% consistent with conjunction
  - 0.3% consistent with material conditional
- Shift of interpretation
  - First three tasks: 38.3% consistent with conditional event
  - Last three tasks: 83.3% consistent with conditional event
  - Strong correlation between conditional event frequency and item position ($r(15) = 0.71$, $p < 0.005$)

Increase of cond. event resp. ($n_1 = 20$) (Pfeifer, 2013a, *Thinking & Reasoning*)



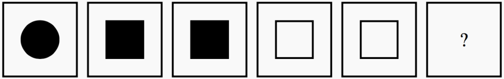
Beyond "abstract" indicative conditionals

Experimental design (Pfeifer & Tulkki, 2017):

	indicative	counterfactual
non-causal	$n_1 = 20$	$n_2 = 20$
causal	$n_3 = 20$	$n_4 = 20$
abductive	$n_5 = 20$	$n_6 = 20$

Sample task: non-causal, indicative (Pfeifer & Tulkki, 2017)

Below are illustrated all the sides of a six-sided die. The sides of the die have two kinds of properties: color (*black* or *white*) and figure (*circle*, *triangle* or *square*).
Question mark means a covered side.



Imagine, that this die is placed in a cup. Then the cup is shaken randomly. Finally, the cup is placed on a table upside down, so that you cannot see which side of the die is facing upwards.

Question: How sure you can be, that the following sentence holds?

If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

Answer:

	at least	*at most*
how many	☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	☐ 0 ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6
out of how many	☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6	☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

Sample task: causal, counterfactual (Pfeifer & Tulkki, 2017)

Here you see patient reports from medical studies concerning three new drugs. Each patient report shows the name of the new drug (*Zotarin*, *Xebutol* or *Raverat*) and its impact (*diminishing symptoms* or *no impact on symptoms*).
Question mark means a covered report.

Zotarin *no impact on symptoms*	*Xebutol* *no impact on symptoms*	*Xebutol* *no impact on symptoms*	*Xebutol* *diminishes symptoms*	*Xebutol* *diminishes symptoms*	?

Imagine a patient, who takes *Xebutol* and view the patient reports again.

Question: How sure you can be, that the following sentence holds?

If the patient were to take *Zotarin*, **then** this would have *no impact* on the symptoms.

counterfactual
= subjunctive mood + factual statement ("who takes *Xebutol*")

Inferentialism and Δp

Inferentialist accounts of conditionals claim that there must be some inferential connection between the antecedent and the consequent of a conditional in order to assert it (see, e.g., Douven, 2016; Douven, Elqayam, Singmannc, & van Wijnbergen-Huitink, 2018; Skovgaard-Olsen, Singmann, & Klauer, 2016).

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The strength of the inferential connection (or "relevance") can be measured by Δp:

$$\Delta p(\text{If } A, \text{ then } C) =_{def.} p(C|A) - p(C|\neg A)$$

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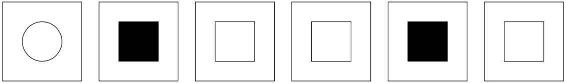
Sample where Δp is violated (Pfeifer & Tulkki, 2017, in prep.)

Alla on kuvattuna kaikki kyljet kuusikylkisestä nopasta. Kylkien kuvioissa on kahdenlaisia ominaisuuksia: väri (*musta* tai *valkoinen*) ja muoto (*ympyrä*, *kolmio*, tai *neliö*).



Kuvittele, että tämä noppa laitetaan kuppiin. Tämän jälkeen kuppia ravistellaan sattumanvaraisesti. Lopuksi kuppi asetetaan pöydälle nurinpäin siten, että et voi nähdä mikä nopan kyljistä osoittaa ylöspäin.

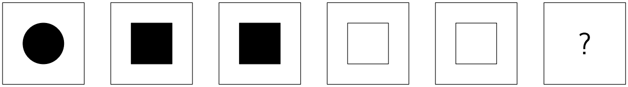
Kysymys: Kuinka varma voit olla siitä, että seuraava lause pitää paikkansa?

If square, then white

Jos ylöspäin osoittavan kyljen kuvio on *neliö*, **niin** tämä kuvio on *valkoinen*.

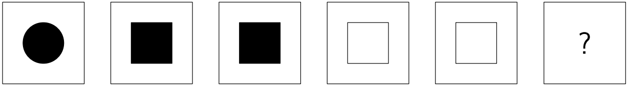
$$\underbrace{p(\text{white}|\text{square})}_{3/5} - \underbrace{p(\text{white}|\neg\text{square})}_{1/1} = -2/5 < 0$$

What is Δp in the context of incomplete probabilistic information?



If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

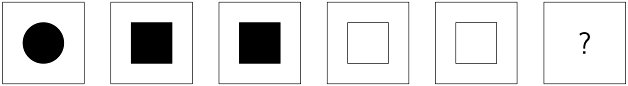
What is Δp in the context of incomplete probabilistic information?



If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$1/2 \leq p(\text{black}|\text{circle}) \leq 2/2$$

What is Δp in the context of incomplete probabilistic information?

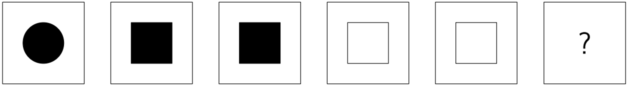


If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$1/2 \leq p(\text{black}|\text{circle}) \leq 2/2$$

$$2/5 \leq p(\text{black}|\neg\text{circle}) \leq 3/5$$

What is Δp in the context of incomplete probabilistic information?



If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

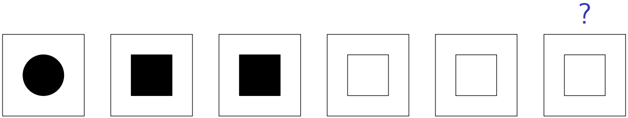
$$1/2 \leq p(\text{black}|\text{circle}) \leq 2/2$$

$$2/5 \leq p(\text{black}|\neg\text{circle}) \leq 3/5$$

The symbol of the covered card may be any one of four possibilities!

What is Δp in the context of incomplete probabilistic information?

Possibility #1:

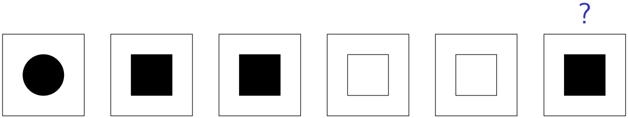


If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$\Delta p_{\text{possibility } \#1} = \underbrace{p(\text{black}|\text{circle})}_{1/1} - \underbrace{p(\text{black}|\neg\text{circle})}_{2/5} = 3/5 > 0$$

What is Δp in the context of incomplete probabilistic information?

Possibility #2:

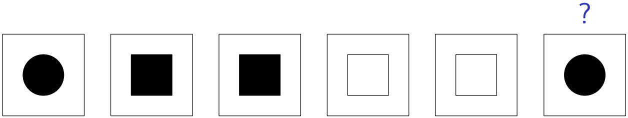


If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$\Delta p_{\text{possibility \#2}} = \underbrace{p(\text{black}|\text{circle})}_{1/1} - \underbrace{p(\text{black}|\neg\text{circle})}_{3/5} = 2/5 > 0$$

What is Δp in the context of incomplete probabilistic information?

Possibility #3:

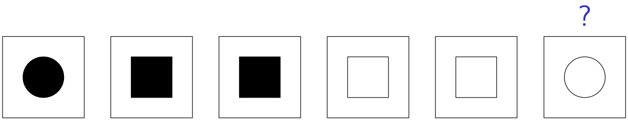


If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$\Delta p_{\text{possibility \#3}} = \underbrace{p(\text{black}|\text{circle})}_{2/2} - \underbrace{p(\text{black}|\neg\text{circle})}_{2/4} = 1/2 > 0$$

What is Δp in the context of incomplete probabilistic information?

Possibility #4:



If the figure on the upward facing side of the die is a *circle*, **then** the figure is *black*.

$$\Delta p_{\text{possibility \#4}} = \underbrace{p(\text{black}|\text{circle})}_{1/2} - \underbrace{p(\text{black}|\neg\text{circle})}_{2/4} = 0$$

Sample Δp-values

task	# ?-info	possible Δp values
T3	1	0.0, 0.4, 0.5, 0.6

Sample Δp-values

task	# ?-info	possible Δp values
T3	1	0.0, 0.4, 0.5, 0.6
T4	3	-1.8, -1.5, -1.3, -1.2, -1.0, -0.8, -0.8, -0.8, -0.8, -0.8, -0.7, -0.7, -0.7, -0.6, -0.6, -0.5, -0.5, -0.5, -0.4, -0.4, -0.4, -0.4, -0.4, -0.4, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.3, -0.2, -0.2, -0.2, -0.2, -0.2, -0.2, -0.2, -0.2, -0.2, -0.1, -0.1, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.1, 0.1, 0.2, 0.2, 0.3, 0.3, 0.3, 0.3, 0.5

	T1	T2	T3	T4	T5	T6
# ?-info	0	0	1	3	1	1
# Δp*-values*	1	1	4	64	4	4
Mean	-0.40	0.50	0.38	-0.32	0.44	0.27
SD	–	–	0.26	0.42	0.18	0.21
Min	–	–	0.00	-1.75	0.25	0.00
Max	–	–	0.60	0.50	0.67	0.50
% $\Delta p > 0$	0	100	75	14	100	75
% $\Delta p = 0$	0	0	25	14	0	25
% $\Delta p < 0$	100	0	0	72	0	0
	T7	T8	T9			
# ?-info	3	2	3			
# Δp*-values*	64	16	64			
Mean	-0.11	0.22	-0.01			
SD	0.40	0.22	0.46			
Min	-1.17	-0.17	-1.50			
Max	0.83	0.60	0.83			
% $\Delta p > 0$	33	81	47			
% $\Delta p = 0$	12	0	17			
% $\Delta p < 0$	55	19	36			

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T1	T2	T3	T4	T5	T6
$[p(\cdot\|\cdot)]$	[48]	[52]	[15]	[16]	[23]	[24]
$[p(\cdot\|\cdot)_{\overline{l}}]$	[- -]	[- -]	[8]	[13]	[17]	[12]
$[p(\cdot\|\cdot)_{\overline{u}}]$	[- -]	[- -]	[19]	[8]	[11]	[10]
$[p(\cdot\|\cdot)_{\overline{lu}}]$	[- -]	[- -]	[1]	[3]	[2]	[1]
Grouped $p(\cdot\|\cdot)$	48	52	43	40	53	47
$p(\cdot \wedge \cdot)$	23	27	34	41	36	32
$p(\cdot \supset \cdot)$	2	0	0	0	0	1
$p(\cdot \equiv \cdot)$	[- -]	[- -]	1	[- -]	[- -]	0
$p(\cdot\|\|\cdot)$	[- -]	[- -]	2	[- -]	[- -]	0
Other	27	22	21	19	12	21

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T1	T2	T3	T4	T5	T6
$[p(\cdot\|\cdot)]$	[48]	[52]	[15]	[16]	[23]	[24]
$[p(\cdot\|\cdot)_{\overline{l}}]$	[- -]	[- -]	[8]	[13]	[17]	[12]
$[p(\cdot\|\cdot)_{\overline{u}}]$	[- -]	[- -]	[19]	[8]	[11]	[10]
$[p(\cdot\|\cdot)_{\overline{lu}}]$	[- -]	[- -]	[1]	[3]	[2]	[1]
Grouped $p(\cdot\|\cdot)$	48	52	43	40	53	47
$p(\cdot \wedge \cdot)$	23	27	34	41	36	32
$p(\cdot \supset \cdot)$	2	0	0	0	0	1
$p(\cdot \equiv \cdot)$	[- -]	[- -]	1	[- -]	[- -]	0
$p(\cdot\|\|\cdot)$	[- -]	[- -]	2	[- -]	[- -]	0
Other	27	22	21	19	12	21
% $\Delta p > 0$	0	100	75	14	100	75

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T7	T8	T9	T10	T11	T12
$[p(\cdot\vert\cdot)]$	[23]	[27]	[25]	[55]	[56]	[29]
$[p(\cdot\vert\cdot)_{\overline{l}}]$	[10]	[13]	[9]	[- -]	[- -]	[10]
$[p(\cdot\vert\cdot)_{\overline{u}}]$	[15]	[7]	[9]	[- -]	[- -]	[18]
$[p(\cdot\vert\cdot)_{\overline{lu}}]$	[0]	[0]	[0]	[- -]	[- -]	[0]
Grouped $p(\cdot\vert\cdot)$	48	46	43	55	56	58
$p(\cdot \wedge \cdot)$	33	31	33	28	28	30
$p(\cdot \supset \cdot)$	0	0	0	1	0	0
$p(\cdot \equiv \cdot)$	[- -]	[- -]	[- -]	[- -]	[- -]	0
$p(\cdot\Vert\cdot)$	[- -]	[- -]	[- -]	[- -]	[- -]	1
Other	18	23	23	17	17	12

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T7	T8	T9	T10	T11	T12
$[p(\cdot\|\cdot)]$	[23]	[27]	[25]	[55]	[56]	[29]
$[p(\cdot\|\cdot)_{\overline{l}}]$	[10]	[13]	[9]	[- -]	[- -]	[10]
$[p(\cdot\|\cdot)_{\overline{u}}]$	[15]	[7]	[9]	[- -]	[- -]	[18]
$[p(\cdot\|\cdot)_{\overline{lu}}]$	[0]	[0]	[0]	[- -]	[- -]	[0]
Grouped $p(\cdot\|\cdot)$	48	46	43	55	56	58
$p(\cdot \wedge \cdot)$	33	31	33	28	28	30
$p(\cdot \supset \cdot)$	0	0	0	1	0	0
$p(\cdot \equiv \cdot)$	[- -]	[- -]	[- -]	[- -]	[- -]	0
$p(\cdot\|\|\cdot)$	[- -]	[- -]	[- -]	[- -]	[- -]	1
Other	18	23	23	17	17	12
% $\Delta p > 0$	33	81	47	0	100	75

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T13	T14	T15	T16	T17	T18
$[p(\cdot\|\cdot)]$	[35]	[35]	[30]	[28]	[32]	[31]
$[p(\cdot\|\cdot)_{\bar{l}}]$	[9]	[13]	[14]	[13]	[17]	[14]
$[p(\cdot\|\cdot)_{\bar{u}}]$	[9]	[8]	[11]	[13]	[7]	[10]
$[p(\cdot\|\cdot)_{\overline{lu}}]$	[0]	[0]	[1]	[2]	[0]	[0]
Grouped $p(\cdot\|\cdot)$	53	56	56	54	55	55
$p(\cdot \wedge \cdot)$	29	30	28	32	26	29
$p(\cdot \supset \cdot)$	0	0	0	0	0	0
$p(\cdot \equiv \cdot)$	[- -]	[- -]	0	[- -]	[- -]	[- -]
$p(\cdot\|\|\cdot)$	[- -]	[- -]	3	[- -]	[- -]	[- -]
Other	18	14	13	14	19	16

Results: responses in percentages ($N = 120$) (Pfeifer & Tulkki, 2017)

Interpretation	T13	T14	T15	T16	T17	T18
$[p(\cdot\|\cdot)]$	[35]	[35]	[30]	[28]	[32]	[31]
$[p(\cdot\|\cdot)_{\overline{l}}]$	[9]	[13]	[14]	[13]	[17]	[14]
$[p(\cdot\|\cdot)_{\overline{u}}]$	[9]	[8]	[11]	[13]	[7]	[10]
$[p(\cdot\|\cdot)_{\overline{lu}}]$	[0]	[0]	[1]	[2]	[0]	[0]
Grouped $p(\cdot\|\cdot)$	53	56	56	54	55	55
$p(\cdot \wedge \cdot)$	29	30	28	32	26	29
$p(\cdot \supset \cdot)$	0	0	0	0	0	0
$p(\cdot \equiv \cdot)$	[- -]	[- -]	0	[- -]	[- -]	[- -]
$p(\cdot\|\|\cdot)$	[- -]	[- -]	3	[- -]	[- -]	[- -]
Other	18	14	13	14	19	16
% $\Delta p > 0$	14	100	75	33	81	47

Percentages of response types in Pfeifer and Stöckle-Schobel (2015) ($N = 80$)

Interpretation	T1	T2	T3	T4	T5	T6
$p(\cdot \supset \cdot)$	0	1	1	0	0	3
$p(\cdot \wedge \cdot)$	5	13	13	10	9	6
$p(\cdot\|\cdot)$	63	74	84	78	81	80
Other	28	12	2	12	10	11

	T7	T8	T9	T10	T11	T12
$p(\cdot \supset \cdot)$	1	1	0	0	1	1
$p(\cdot \wedge \cdot)$	10	8	8	6	8	8
$p(\cdot\|\cdot)$	83	79	86	86	89	85
Other	6	12	6	8	2	6

	T13	T14	T15	T16	T17	T18
$p(\cdot \supset \cdot)$	0	1	1	1	0	0
$p(\cdot \wedge \cdot)$	8	8	6	8	5	5
$p(\cdot\|\cdot)$	85	88	89	78	83	90
Other	7	3	4	13	12	5

	T19
$p(\cdot \supset \cdot)$	3
$p(\cdot \wedge \cdot)$	5
$p(\cdot\|\cdot)$	86
Other	6

Percentages of response types in Pfeifer and Stöckle-Schobel (2015) ($N = 80$)

Interpretation	T1	T2	T3	T4	T5	T6
$p(\cdot \supset \cdot)$	0	1	1	0	0	3
$p(\cdot \wedge \cdot)$	5	13	13	10	9	6
$p(\cdot\vert\cdot)$	63	74	84	78	81	80
Other	28	12	2	12	10	11
Δp	0.33	-0.80	-0.20	-0.75	0.00	0.00
	T7	T8	T9	T10	T11	T12
$p(\cdot \supset \cdot)$	1	1	0	0	1	1
$p(\cdot \wedge \cdot)$	10	8	8	6	8	8
$p(\cdot\vert\cdot)$	83	79	86	86	89	85
Other	6	12	6	8	2	6
Δp	0.33	-0.25	0.25	0.33	0.25	-0.80
	T13	T14	T15	T16	T17	T18
$p(\cdot \supset \cdot)$	0	1	1	1	0	0
$p(\cdot \wedge \cdot)$	8	8	6	8	5	5
$p(\cdot\vert\cdot)$	85	88	89	78	83	90
Other	7	3	4	13	12	5
Δp	0.00	0.75	-0.75	0.00	0.00	0.25
	T19					
$p(\cdot \supset \cdot)$	3					
$p(\cdot \wedge \cdot)$	5					
$p(\cdot\vert\cdot)$	86					
Other	6					
Δp	-0.20					

Percentages of response types in Pfeifer and Stöckle-Schobel (2015) ($N = 80$)

Interpretation	T1	T2	T3	T4	T5	T6
$p(\cdot \supset \cdot)$	0	1	1	0	0	3
$p(\cdot \wedge \cdot)$	5	13	13	10	9	6
$p(\cdot\|\cdot)$	63	74	84	78	81	80
Other	28	12	2	12	10	11
Δp	0.33	-0.80	-0.20	-0.75	0.00	0.00
	T7	T8	T9	T10	T11	T12
$p(\cdot \supset \cdot)$	1	1	0	0	1	1
$p(\cdot \wedge \cdot)$	10	8	8	6	8	8
$p(\cdot\|\cdot)$	83	79	86	86	89	85
Other	6	12	6	8	2	6
Δp	0.33	-0.25	0.25	0.33	0.25	-0.80
	T13	**T14**	**T15**	T16	T17	T18
$p(\cdot \supset \cdot)$	0	1	1	1	0	0
$p(\cdot \wedge \cdot)$	8	8	6	8	5	5
$p(\cdot\|\cdot)$	**85**	**88**	**89**	78	83	90
Other	7	3	4	13	12	5
Δp	**0.00**	**0.75**	**-0.75**	0.00	0.00	0.25
	T19					
$p(\cdot \supset \cdot)$	3					
$p(\cdot \wedge \cdot)$	5					
$p(\cdot\|\cdot)$	86					
Other	6					
Δp	-0.20					

Further results

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  but (incorrectly) $p(x = 2 \vee x = 4|x = 2) = 0$ (Fugard, Pfeifer, & Mayerhofer, 2011)

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From modus ponens to generalised modus ponens

	Modus ponens	Generalised modus ponens
(Categorical premise)	A	$A\|H$
(Conditional premise)	If A, then C	If $A\|H$, then C
(Conclusion)	C	C

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Sample instantiation (Gibbard, 1981, p. 237):

$\overbrace{\textit{The cup breaks if dropped}}^{A|H}$.

If $\overbrace{\textit{the cup breaks if dropped}}^{A|H}$, then $\overbrace{\textit{the cup is fragile}}^{C}$.

Therefore, $\overbrace{\textit{the cup is fragile}}^{C}$.

Generalised Probabilistic MP (Sanfilippo, Pfeifer, & Gilio, 2017)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}(C\|(A\|H)) = y$
C	$? \leq p(C) \leq ?$

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In betting terms, $\mu = \mathbb{P}[C \mid (A \mid H)]$ represents the amount you agree to pay, with the proviso that you will receive the quantity:

$$C \mid (A \mid H) = \begin{cases} 1, & \text{if } A \wedge H \wedge C \text{ true}, \\ 0, & \text{if } A \wedge H \wedge \neg C \text{ true}, \\ \mu, & \text{if } \neg A \wedge H \text{ true}, \\ x + \mu(1 - x), & \text{if } \neg H \wedge C \text{ true}, \\ \mu(1 - x), & \text{if } \neg H \wedge \neg C \text{ true}. \end{cases}$$

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Since $(C|A)|H \neq C|(A \wedge H)$, the Import-Export Principle does not hold. Thus, Lewis' first triviality result (1976) is avoided (Gilio & Sanfilippo, 2014).

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A\|H$	$p(A\|H) = x$
If $A\|H$, then C	$\mathbb{P}(C\|(A\|H)) = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Generalised modus ponens (Sanfilippo, Pfeifer, & Gilio, 2017, Theorem 5, p. 487)

Generalised modus ponens	Generalised probabilistic modus ponens
$A \vert H$	$p(A \vert H) = x$
If $A \vert H$, then C	$\mathbb{P}(C \vert (A \vert H)) = y$
C	$? \leq p(C) \leq ?$

How do we propagate the uncertainty from the premises to the conclusion?

Theorem

Given any coherent assessment (x, y) on $\{A|H, C|(A|H)\}$, with A, C, H logically independent, but $A \neq \bot$ and $H \neq \bot$. The conclusion $p(C)$ is coherent iff

$$xy \leq p(C) \leq xy + 1 - x$$

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which are just the same probability propagation rules as in the non-nested probabilistic modus ponens. (I.e., from $p(A) = x$ and $p(C \mid A) = y$ infer $xy \le P(C) \le xy + 1 - x$.)

Data of the PTTT revisited

Most people interpret their beliefs in conditionals by $p(C|A)$ even if

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Why does conditional probability predict counterfactuals?

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Why does conditional probability predict counterfactuals?

Formally (see, e.g. Gilio & Sanfilippo, 2013),

$$\overbrace{\textit{Prevision}\ \underbrace{[(C|A)|\overbrace{\neg A}^{\text{fact}}]}_{\text{cond. random quantity}}}^{\text{belief in counterfactual}} = \overbrace{\textit{Probability}\ (\underbrace{C|\overbrace{A}^{\text{assumed}}}_{cond.event})}^{\text{belief in indicative conditional}}.$$

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Aristotle's Theses

AT #1: $\neg(\neg A \rightarrow A)$

AT #2: $\neg(A \rightarrow \neg A)$

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AT #1: $\neg(\neg A \to A)$

$$\neg(\neg A \supset A)$$

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AT #1: $\neg(\neg A \to A)$

$$\neg(\neg A \supset A) \quad \equiv \quad \neg A \wedge \neg A \quad \equiv \quad \neg A$$

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$$\neg(A \supset \neg A) \quad \equiv \quad A \wedge A \quad \equiv \quad A$$

Aristotle's Theses: Prob. log. predictions (Pfeifer, 2012a, *The Monist*)

AT #1: $\neg(\neg A \to A)$

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AT #2: $\neg(A \rightarrow \neg A)$

- $P(\neg(A \supset \neg A)) = P(A)$
- $P(\neg A|A) = 0$, its negation: $P(\neg\neg A|A) = P(A|A) = 1$

Experiment 1: Abstract version, Aristotle's Thesis #1

The letter "*A*" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

- "*A* and not-*A*" is guaranteed to be false.
- "*A* or not-*A*" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "*A*" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

Evaluate the following sentence (please tick exactly one alternative):

It is not the case, that: If not-*A*, then *A*.

The sentence in the box is guaranteed to be false	□
The sentence in the box is guaranteed to be true	□
One cannot infer whether the sentence is true or false	□

Experiment 1: Abstract version, Aristotle's Thesis #2

The letter "*A*" denotes a sentence, like "It is raining".

There are sentences, where you can infer only on the basis of their logical form, whether they are guaranteed to be false or guaranteed to be true. For example:

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- "*A* or not-*A*" is guaranteed to be true.

There are sentences, where you cannot infer only on the basis of their logical form, whether they are true or false. The sentence "*A*" ("It is raining."), for example, can be true but it can just as well be false: this depends upon whether it is actually raining.

Evaluate the following sentence (please tick exactly one alternative):

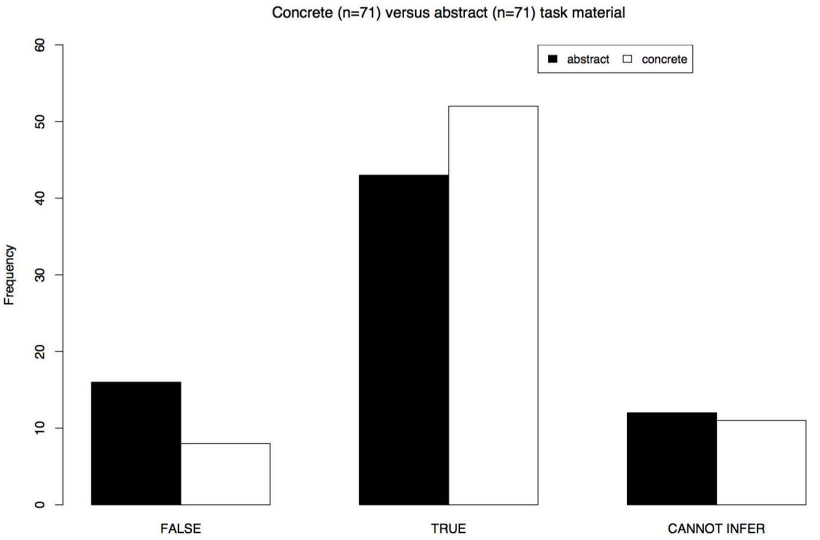
It is not the case, that: If *A*, then not-*A*.

The sentence in the box is guaranteed to be false	☐
The sentence in the box is guaranteed to be true	☐
One cannot infer whether the sentence is true or false	☐

Experiment 1: Sample (Pfeifer, 2012a, *The Monist*)

- $N = 141$
- all psychology students (University of Salzburg)
- 91% third semester
- 78% female
- median age: 21 (1st Qu. = 20, 3rd Qu. =23)

Aristotle's Thesis: Results (Pfeifer, 2012a, *The Monist*. Figure 2)



Scope ambiguities (Pfeifer, 2012a, *The Monist*)

(W) Negating the conditional: $\neg \underbrace{(A \rightarrow \neg A)}_{\text{wide scope}}$

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(W) and (N) are well defined for $\wedge$ and $\supset$. Conditional events, $B|A$, are usually negated by (N), $P(\neg B|A)$.

Experiment 2: Design (Pfeifer, 2012a, *The Monist*)

Between participants: Explicit ($n_1 = 20$) vs. implicit negation ($n_2 = 20$)
Within participants: 12 Tasks

Task	Name	Argument form
1	Aristotle's Thesis 1	$\neg(A \to \neg A)$
2	Negated Reflexivity	$\neg(A \to A)$
3	Aristotle's Thesis 2	$\neg(\neg A \to A)$
4	Reflexivity	$A \to A$
5	Contingent Arg. 1	$A \to B$
6	Contingent Arg. 2	$\neg(A \to B)$
7-10	4 Probabilistic truth-table tasks	
11	Paradox 1	from B infer $A \to B$
12	Neg. Paradox 1	from B infer $A \to \neg B$

Experiment 2: Predictions (Pfeifer, 2012a, *The Monist*)

Argument form	Scope			
		wide	narrow	
	$\cdot\mid\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \rightarrow \neg A)$	T	CT	T	T
$\neg(A \rightarrow A)$	F	F	CT	CT
$\neg(\neg A \rightarrow A)$	T	CT	T	T
$A \rightarrow A$	T	T	T	CT
$A \rightarrow B$	CT	CT	CT	CT
$\neg(A \rightarrow B)$	CT	CT	CT	CT
from B infer $A \rightarrow B$	U	H		U
from B infer $A \rightarrow \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Experiment 2: Predictions $\cdot|\cdot$ against <u>wide</u> scope of $\cdot \supset \cdot$

Argument form	Scope			
		wide	narrow	
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \rightarrow \neg A)$	T	CT	T	T
$\neg(A \rightarrow A)$	F	F	CT	CT
$\neg(\neg A \rightarrow A)$	T	CT	T	T
$A \rightarrow A$	T	T	T	CT
$A \rightarrow B$	CT	CT	CT	CT
$\neg(A \rightarrow B)$	CT	CT	CT	CT
from B infer $A \rightarrow B$	U	H		U
from B infer $A \rightarrow \neg B$	U	H		L

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Experiment 2: Predictions $\cdot|\cdot$ against narrow scope of $\cdot \supset \cdot$

Argument form		Scope		
		wide	narrow	
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$
$\neg(A \to \neg A)$	T	CT	T	T
$\neg(A \to A)$	F	F	CT	CT
$\neg(\neg A \to A)$	T	CT	T	T
$A \to A$	T	T	T	CT
$A \to B$	CT	CT	CT	CT
$\neg(A \to B)$	CT	CT	CT	CT
from B infer $A \to B$	U	H		U
from B infer $A \to \neg B$	U	H		L

Note: CT=can't tell, T=true, F=false,

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Experiment 2: Sample (Pfeifer, 2012a, *The Monist*)

- $N = 40$ (University of Salzburg)
- no psychology students
- individual tested
- 50% female
- median age: 22 (1st Qu. = 21, 3rd Qu. =23)

Experiment 2: Results (Pfeifer, 2012a, *The Monist*)

Argument form		Scope wide	Scope narrow		Responses in percent T	F	CT
	$\cdot\vert\cdot$	$\cdot \supset \cdot$	$\cdot \supset \cdot$	$\cdot \wedge \cdot$	T	F	CT
$\neg(A \to \neg A)$	T	CT	T	T	78	18	5
$\neg(A \to A)$	F	F	CT	CT	10	88	2
$\neg(\neg A \to A)$	T	CT	T	T	80	13	8
$A \to A$	T	T	T	CT	93	3	5
$A \to B$	CT	CT	CT	CT	0	13	88
$\neg(A \to B)$	CT	CT	CT	CT	20	3	78
from B infer $A \to B$	U	H		U	40	0	60
from B infer $A \to \neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

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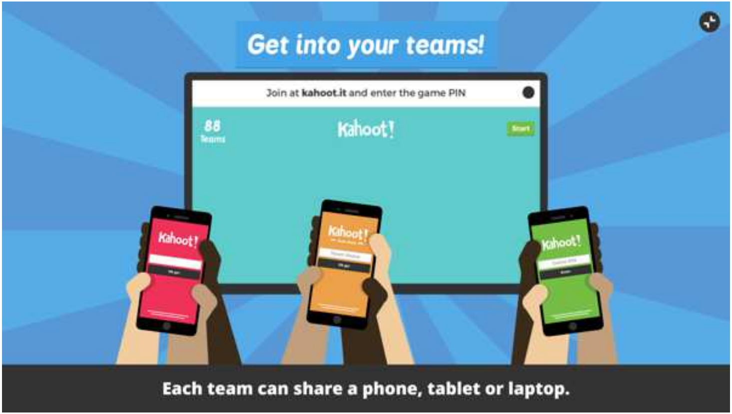
Experiment 2: Results (Pfeifer, 2012a, *The Monist*)

Argument form		Scope			Responses in percent		
		wide	narrow				
	$\cdot\mid\cdot$	$\cdot\supset\cdot$	$\cdot\supset\cdot$	$\cdot\wedge\cdot$	T	F	CT
$\neg(A \to \neg A)$	T	CT	T	T	78	18	5
$\neg(A \to A)$	F	F	CT	CT	10	88	2
$\neg(\neg A \to A)$	T	CT	T	T	80	13	8
$A \to A$	T	T	T	CT	93	3	5
$A \to B$	CT	CT	CT	CT	0	13	88
$\neg(A \to B)$	CT	CT	CT	CT	20	3	78
from B infer $A \to B$	U	H		U	40	0	60
from B infer $A \to \neg B$	U	H		L	5	30	65

Note: CT=can't tell, T=true, F=false,

U=uninformative conclusion probability, H=high conclusion probability, L=low conclusion probability

Time for a quiz!



. . . and go to

kahoot.it

Other connexive principle: Aristotle's Second Thesis

$$\neg((A \to B) \wedge (\neg A \to B))$$

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$p(B|A)$ does not constrain $p(B|\neg A)$ and *vice versa*. Therefore, Aristotle's Second Thesis does not hold.

Also in the theory of conditional random quantities, the prevision in $\neg((B|A) \wedge (B|\neg A))$ is not in general equal to 1.

Connexive principle: Boethius' theses

(BT1) $(A \to B) \to \neg(A \to \neg B)$

(BT2) $(A \to \neg B) \to \neg(A \to B)$

Connexive principle: Boethius' theses

(BT1) $(A \to B) \to \neg(A \to \neg B)$

(BT2) $(A \to \neg B) \to \neg(A \to B)$

Both versions of Boethius' theses hold under the narrow scope negation (e.g., for (BT1) note that $\neg\neg B|A = B|A$).

Connexive principle: Abelard's First Principle

$$\neg((A \rightarrow B) \wedge (A \rightarrow \neg B))$$

Connexive principle: Abelard's First Principle

$$\neg((A \to B) \wedge (A \to \neg B))$$

If $p(B|A) = x$, then, by coherence $p(\neg B|A) = 1 - x$. Since, in general $p(B|A) + p(\neg B|A) = 1$, it cannot be the case that both, $p(B|A)$ and $p(\neg B|A)$ are "high" (i.e., $> .5$) Therefore, Abelard's First Principle holds.

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Within the theory of conditional random quantities, we observe that:

$$(B|A) \wedge (\neg B|A) = \bot|A$$

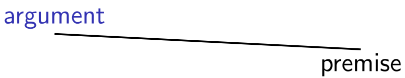
The only coherent assessment of $\bot|A$ is 0. Therefore, Abelard's First Principle holds.

Table of contents

What is argument strength?

argument

What is argument strength?



What is argument strength?

argument

⟨ premise(s) , conclusion ⟩ premise

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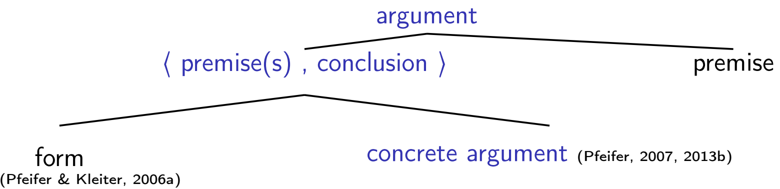

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form
(Pfeifer & Kleiter, 2006a)

concrete argument (Pfeifer, 2007, 2013b)

uncertain consequence relation
Bayes' theorem
(e.g. Hahn & Oaksford, 2006)

dynamic

ignores the structure premises: e.g.,
how shall we assess our degree of belief in

$$\overbrace{C}^{\text{conclusion}} \;|\; \overbrace{(A \text{ and } (C|A))}^{\text{premises}} \quad ?$$

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measures of confirmation (see Crupi, Tentori, & Gonzales, 2007):

$D(e,h) = p(h\|e) - p(h)$	(Carnap, 1962)
$S(e,h) = p(h\|e) - p(h\|\neg e)$	(Christensen, 1999)
$M(e,h) = p(e\|h) - p(e)$	(Mortimer, 1988)
$N(e,h) = p(e\|h) - p(e\|\neg h)$	(Nozick, 1981)
$C(e,h) = p(e \wedge h) - p(e) \times p(h)$	(Carnap, 1962)
$R(e,h) = [p(h\|e)/p(h)] - 1$	(Finch, 1960)
$G(e,h) = 1 - [p(\neg h\|e)/p(\neg h)]$	(Rips, 2001)
$L(e,h) = \frac{p(e\|h) - p(e\|\neg h)}{p(e\|h) + p(e\|\neg h)}$	(Kemeny & Oppenheim, 1952)

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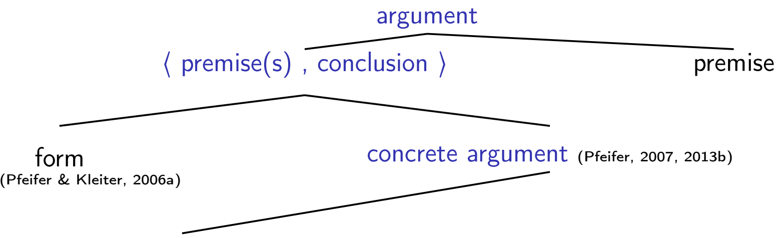

uncertain consequence relation
measures of confirmation as argument strength

$\mathfrak{s}_{D(\mathcal{P},\mathcal{C})} = p(\mathcal{C}\|\mathcal{P}) - p(\mathcal{C})$	(Carnap, 1962)
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ignores the structure of the premises

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Idea: An argument ist strong iff its conclusion probability is high and precise

Measuring argument strength (Pfeifer, 2013b)

Let x' and x'' denote the tightest coherent lower and upper probability bounds of the conclusion $\mathcal{C}$ of an argument $\mathcal{A}$, respectively.

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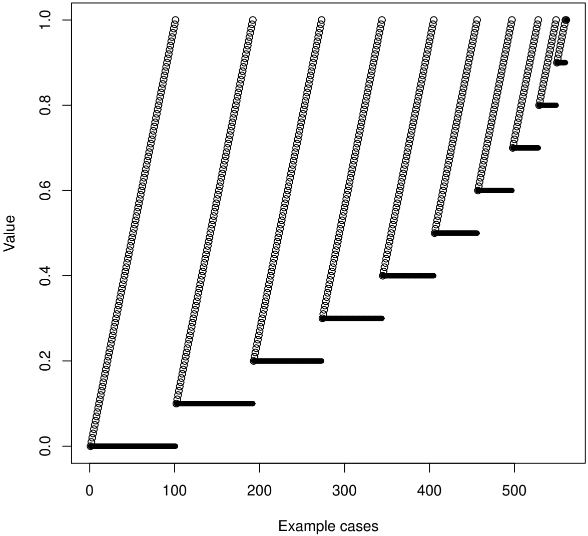
The argument strength $\mathfrak{s}$ is defined by

$$\mathfrak{s} =_{\text{def.}} \overbrace{(1-(x''-x'))}^{\text{precision}} \times \overbrace{\frac{x'+x''}{2}}^{\text{location}},$$

where $0 \leq \mathfrak{s} \leq 1$, and 0 equals minimum and 1 equals maximum argument strength.

Strength: $\mathfrak{s} = (1 - (x'' - x')) \times ((x' + x'')/2)$ (Pfeifer, 2013b)



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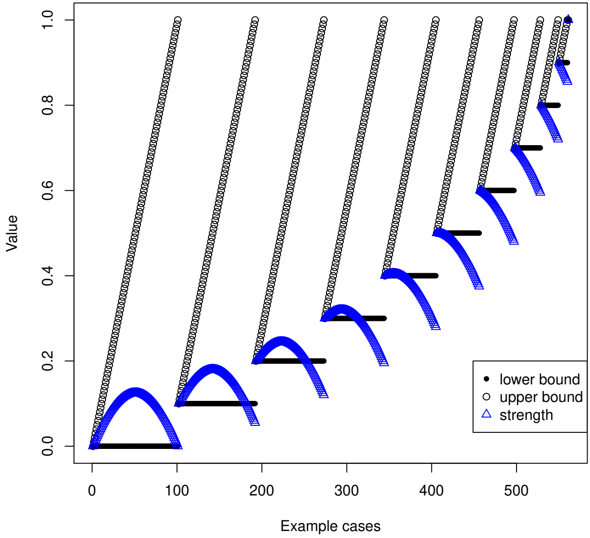

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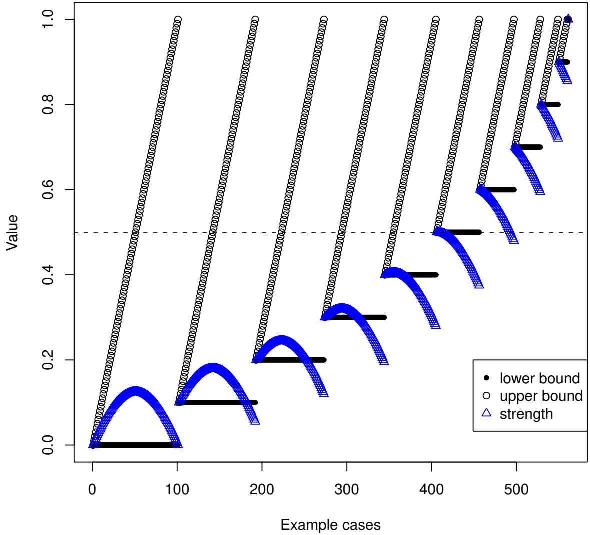

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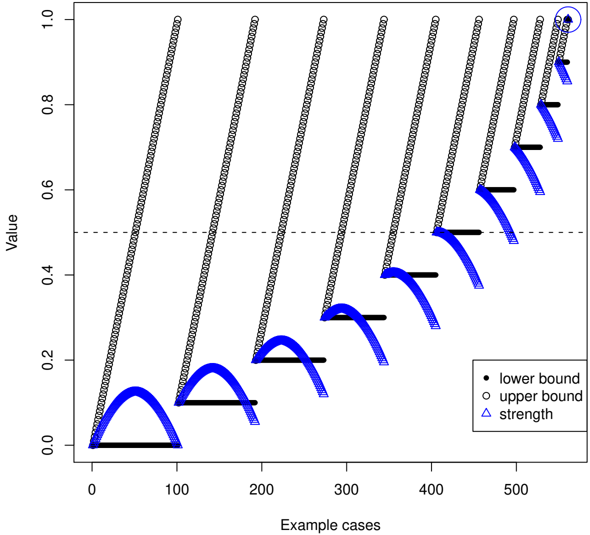

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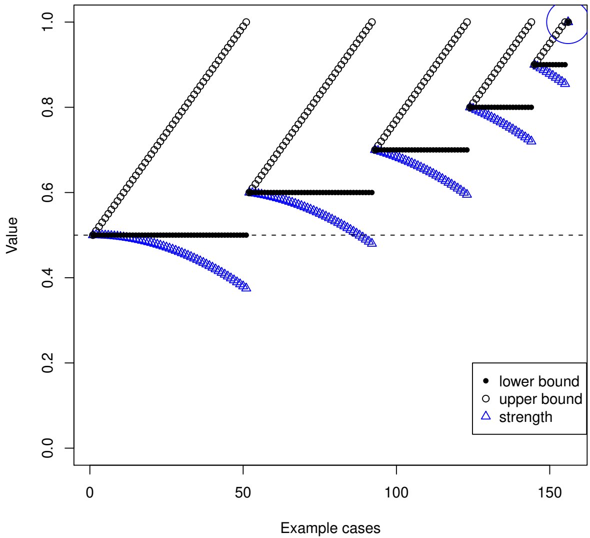
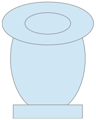


Ellsberg paradox (Ellsberg, 1961, p. 653f)



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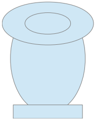

30 red balls; 60 black or yellow balls

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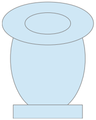

	30 red balls; 60 black or yellow balls
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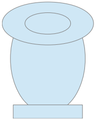

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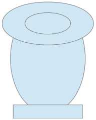

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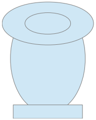

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(a) $\succ$ (b) and (d) $\succ$ (c)

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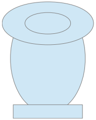
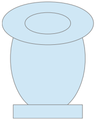


30 red balls; 60 black or yellow balls

(a)	\$100 if red, \$0 otherwise	$p(R) = .33$
(b)	\$100 if black, \$0 otherwise	$0 \leq p(B) \leq .67$
(c)	\$100 if red or yellow, \$0 otherwise	$.33 \leq p(R \vee Y) \leq 1$
(d)	\$100 if black or yellow, \$0 otherwise	$p(B \vee Y) = .67$

$$(a) \succ (b) \quad \text{and} \quad (d) \succ (c)$$

Ellsberg paradox (Ellsberg, 1961, p. 653f)



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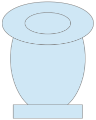

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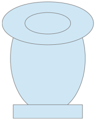
(a) $\succ$ (b) and (d) $\succ$ (c)

If $p(R) > p(B)$, then $p(B \vee Y) < p(R \vee Y)$

Ellsberg paradox (Ellsberg, 1961, p. 653f)



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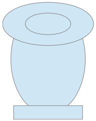

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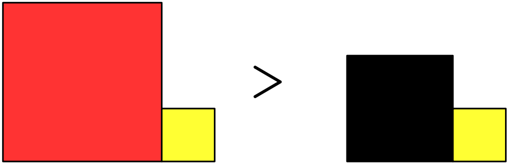
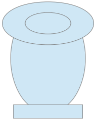


Ellsberg paradox — epistemic version (Pfeifer & Pankka, 2017)



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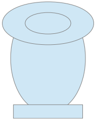

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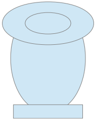

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$\mathcal{A}_1$ for (a) $\quad$ $\mathcal{A}_2$ for (b) $\quad$ $\mathcal{A}_3$ for (c) $\quad$ $\mathcal{A}_4$ for (d)

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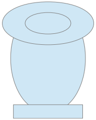

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$\mathcal{A}_1$ for (a)	$\mathcal{A}_2$ for (b)	$\mathcal{A}_3$ for (c)	$\mathcal{A}_4$ for (d)
$\mathfrak{s}(\mathcal{A}_1)$=.33	$\mathfrak{s}(\mathcal{A}_2) = .11$	$\mathfrak{s}(\mathcal{A}_3) = .22$	$\mathfrak{s}(\mathcal{A}_4) = .67$

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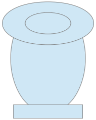

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$\mathcal{A}_1$ for (a)	$\mathcal{A}_2$ for (b)	$\mathcal{A}_3$ for (c)	$\mathcal{A}_4$ for (d)
$\mathfrak{s}(\mathcal{A}_1)$=.33	$\mathfrak{s}(\mathcal{A}_2) = .11$	$\mathfrak{s}(\mathcal{A}_3) = .22$	$\mathfrak{s}(\mathcal{A}_4) = .67$

Measure $\mathfrak{s}$ matches the data (Pfeifer & Pankka, 2017):

$$\mathfrak{s}(\mathcal{A}_1) > \mathfrak{s}(\mathcal{A}_2) \quad \text{and} \quad \mathfrak{s}(\mathcal{A}_4) > \mathfrak{s}(\mathcal{A}_3)$$

Experiment

Sample:

- 60 students (University of Helsinki)
- none of them studied psychology, mathematics, statistics, or philosophy
- 15 € compensation for participation
- individual testing

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- none of them studied psychology, mathematics, statistics, or philosophy
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Design:

Presented probabilities	Formulation	
	epistemic	persuasive
Premise & conclusion	$n_1 = 10$	$n_2 = 10$
Conclusion only	$n_3 = 10$	$n_4 = 10$
Premise only	$n_5 = 10$	$n_6 = 10$

Task material (Argument ranking task)

You will be presented with two arguments. Your task will be to tell, which one is stronger.

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There is an urn that contains 90 balls, of which 30 are red and 60 are black or yellow. The ratio of the black and yellow balls is unknown—there may be from 0 to 60 black (or yellow) balls. One ball is drawn from the urn and you are asked to choose a bet between two options. **Bet 1** means that you will win $100, if the ball drawn from the urn is red. **Bet 2** means that you will win $100, if the ball is black.

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Two of your friends are arguing about which bet you should choose. They both give you an argument.

Task material (Argument ranking task, epistemic condition)

Argument 1 *for* **Bet 1**

I am % sure that the ball drawn from the urn is red.
I am % sure that the ball drawn from the urn is black or yellow.
Therefore, I am 33 % sure that the ball drawn from the urn is red.

Task material (Argument ranking task, epistemic condition)

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Argument 2 *for* **Bet 2**

I am % sure that the ball drawn from the urn is red.
I am % sure that the ball drawn from the urn is black or yellow.
Therefore, I am at least 0 % and at most 67 % sure that the ball drawn from the urn is black.

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Question: Which argument is stronger to know which bet to choose? Tick a box.

□ **Argument 1** □ **Argument 2**

Task material (Argument ranking task, persuasive condition)

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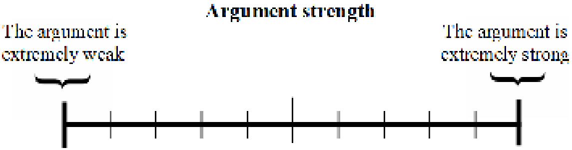

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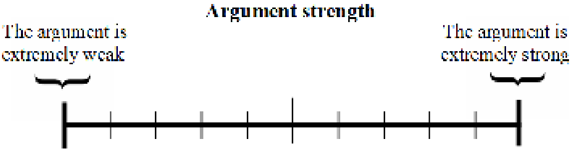

Structure of booklets

1. Introduction of task material
2. Argument ranking tasks
3. Argument rating tasks
4. (original) Ellsberg tasks

Results

- no significant differences among the groups (epistemic/persuasive, presented percentages)
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%	arg. ranking	Ellsberg	%	arg. ranking	Ellsberg
Bet1	73,3	93,3	Bet3	25,0	23,3
Bet2	26,7	6,7	Bet4	75,0	76,7

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Table: Means and standard deviations (*SD*) of the argument strength ratings $\mathfrak{s}(\cdot)$ on a scale from 0 (“extremely weak”) to 10 (“extremely strong”; $N = 60$).

	$\mathfrak{s}(\mathcal{A}_1)$	$\mathfrak{s}(\mathcal{A}_2)$	$\mathfrak{s}(\mathcal{A}_3)$	$\mathfrak{s}(\mathcal{A}_4)$
Mean	5,20	3,98	5,77	6,95
SD	2,64	2,58	1,74	1,87

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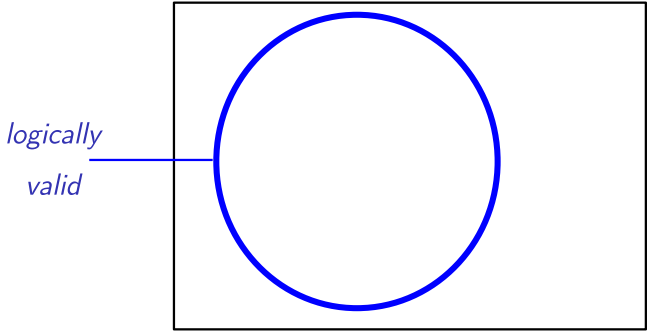
- An argument is probabilistically informative if and only if it is possible that the premise probabilities constrain the conclusion probability. I.e., if the coherent probability interval of its conclusion is not necessarily equal to the unit interval $[0, 1]$ (Pfeifer & Kleiter, 2006a).

Log. valid–prob. informative (Pfeifer & Kleiter (2009). *Journal of Applied Logic*. Figure 1)



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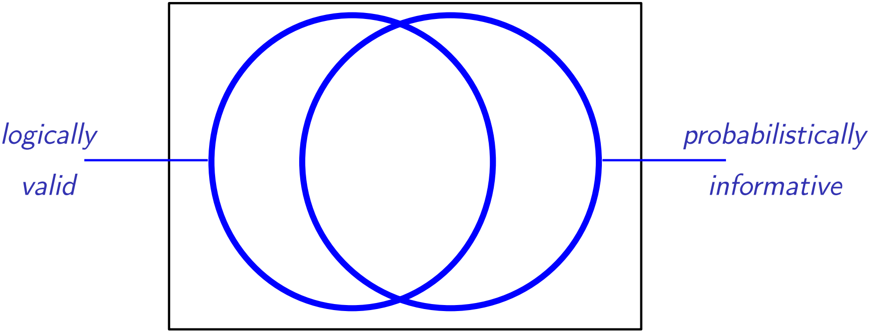

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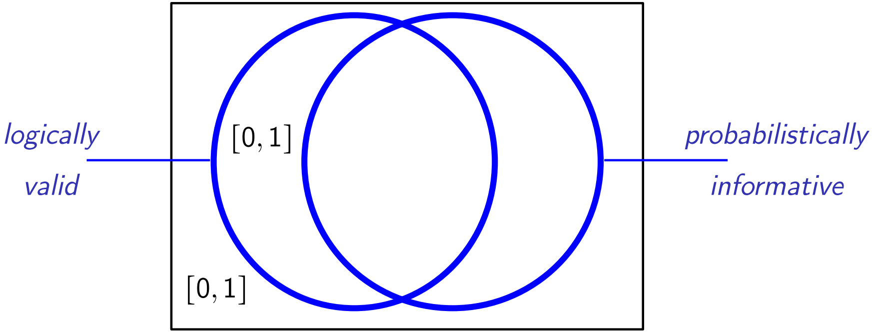

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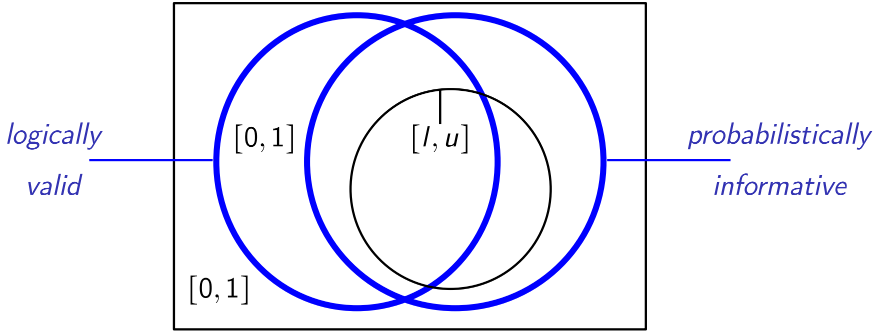

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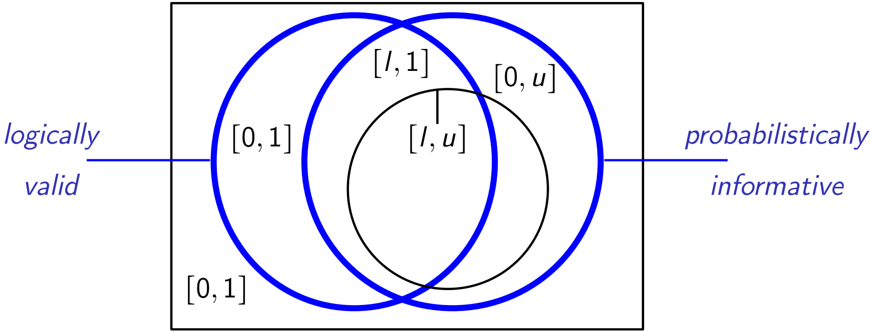

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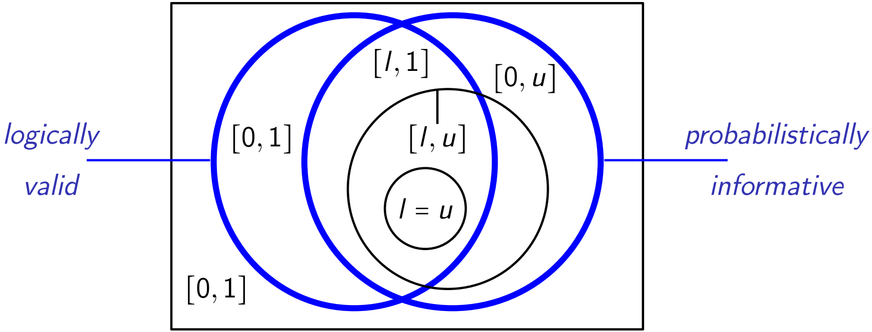

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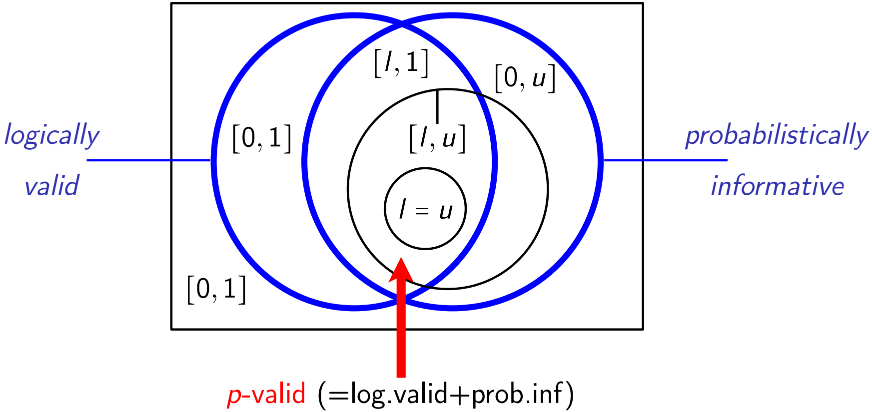

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Motivation

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- Developing coherence based probability logic semantics for Aristotelian syllogisms

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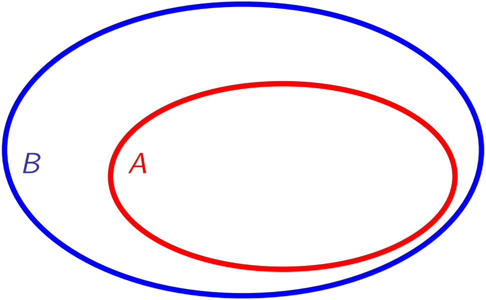

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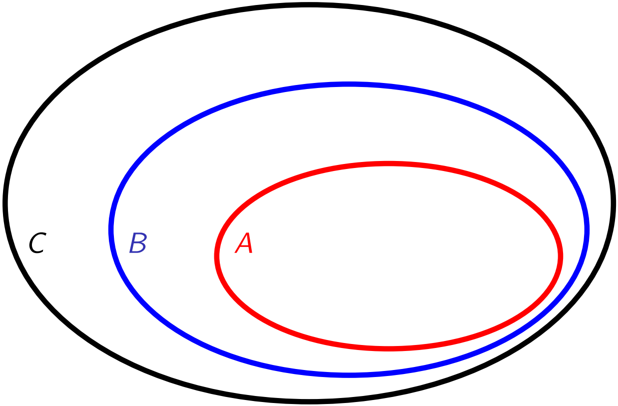

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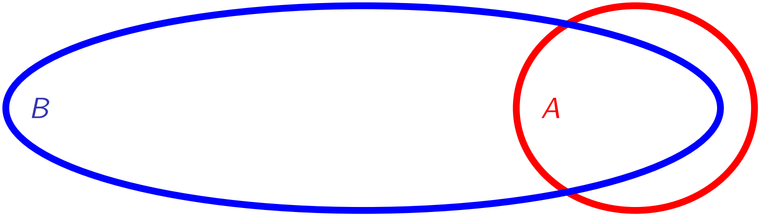

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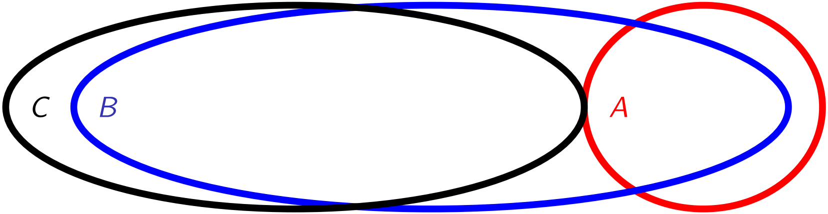

Selected forms of transitivity & empirical evidence

Name	*Formalization*
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- Experimental result: Non-probabilistic tasks: endorsement rate of 89–100% (Evans et al., 1993); probabilistic tasks: 63%-100% coherent responses (Pfeifer & Kleiter, 2007)

Syllogistic types of sentences and figures

Name of Proposition Type		PL formula
(A)	*Universal affirmative*	$\forall x(Sx \supset Px) \wedge \exists x Sx$
(I)	*Particular affirmative*	$\exists x(Sx \wedge Px)$
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	Figure name			
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Premise 1	*MP*	*PM*	*MP*	*PM*
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256 possible syllogisms, 24 Aristotelianly-valid, 9 require $\exists x Sx$

Traditionally valid syllogisms (see, e.g., Pfeifer, 2006a, Figure 2)

	Explicit existence assumptions		Implicit existence assumptions	
Figure I	AAA	Barbara	AAI	Barbari
	AII	Darii	EAO	Celaront
	EAE	Celarent		
	EIO	Ferio		
Figure II	AEE	Camestres	AEO	Camestrop
	AOO	Baroco	EAO	Cesaro
	EAE	Cesare		
	EIO	Festino		
Figure III	AII	Datisi	AAI	Darapti
	EIO	Ferison	EAO	Felapton
	IAI	Disamis		
	OAO	Bocardo		
Figure IV	AEE	Camenes	AAI	Bramantip
	EIO	Fresison	AEO	Camenop
	IAI	Dimaris	EAO	Fesapo

Example: Syllogism

(A)	All philosophers are mortal.
(A)	All members of the Vienna Circle are philosophers.
(A)	All members of the Vienna Circle are mortal.

Modus Barbara

(A)	All M are P
(A)	All S are M
(A)	All S are P

Modus Barbara

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$$\begin{array}{lll} \text{(A)} & \forall x(Mx \supset Px) & (\wedge \exists x Mx) \\ \text{(A)} & \forall x(Sx \supset Mx) & (\wedge \exists x Sx) \\ \hline \text{(A)} & \forall x(Sx \supset Px) & \end{array}$$

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Premise 1	MP	PM	MP	PM
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…transitive structure of Figure 1

Modus Barbari

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Modus Barbari

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\hline
\text{(A)} & \exists x(Sx \wedge Px) &
\end{array}$$

Modus Darii

(A)	All M are P
(I)	At least one S is M
(I)	At least one S is P

(A)	$\forall x(Mx \supset Px)$	$(\wedge \exists x Mx)$
(I)	$\exists x(Sx \wedge Mx)$	$(\wedge \exists x Sx)$
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Previous work: Johann-Heinrich Lambert

Source: Wikimedia Commons
http://tinyurl.com/lbjcruu

*1728 in Mulhouse, former exclave of Switzerland (now Alsace, France) †1777 in Berlin

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Important contributions to

- mathematics (e.g., proof that π is irrational)
- physics (particularly optics), astronomy and map projections
- philosophy
  - distinction between subjective and objective appearances
  - influenced, among others, I. Kant and J. S. Mill
  - logic (syllogisms)

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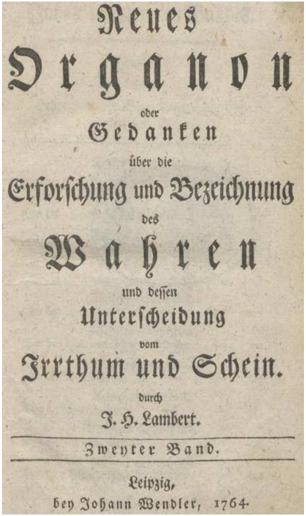

Neues
Organon
oder
Gedanken
über die
Erforschung und Bezeichnung
des
Wahren
und dessen
Unterscheidung
vom
Irrthum und Schein.
durch
J. H. Lambert.
Zweyter Band.
Leipzig,
bey Johann Wendler, 1764.

Source: DTA:SUB Göttingen, 8 PHIL II, 1905:2
http://tinyurl.com/ldpuc5c

Previous work: Johann-Heinrich Lambert (1764)

§. 189. Man habe nun zween Sätze
$\frac{3}{4}$ A sind B
C ist A.

We have now two sentences (p. 358f)
exactly $\frac{3}{4}$ of all A have predicate B
C is an individuum which is A

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[...]
Wenn man demnach den Schluß zieht, daß C, B sey, so ist dieser Schluß nicht völlig gewiß, sondern es geht ihm $\frac{1}{4}$ an der Gewißheit ab, das will sagen, seine Wahrscheinlichkeit ist $\frac{3}{4}$. Dieses drücken wir nun folgendermaßen aus:

We have now two sentences (p. 358f)
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If one draws an inference based on this, that C were B, then this inference is not completely certain, rather it lacks $\frac{1}{4}$ certainty. This means its probability is $\frac{3}{4}$. We express this now as follows:

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[. . .] ſo merken wir an, daß der zwiſchen das Bindwörtgen iſt und das Prädicat B geſetzte Bruch, nicht das Prädicat, ſondern das Bindwörtgen angehe. [. . .] es ſey, daß man ihn vorſetze oder anhänge.

we note, that the fraction between the copula "is" and the predicate B does not relate to the predicate, but to the copula [. . .] it is pre- or postfixed.

Previous work: Johann-Heinrich Lambert (1764)

§. 190. [...]	(p. 359)
$\frac{3}{4}$ A sind B	Exactly $\frac{3}{4}$ of all A have predicate B
C ist A	C is an individuum which is A
folglich C $\frac{3}{4}$ ist B.	Therefore, C ($\frac{3}{4}$ is) B.

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§. 190. […]	(p. 359)
$\frac{3}{4}$ A ſind B	Exactly $\frac{3}{4}$ of all A have predicate B
C iſt A	C is an individuum which is A
folglich C $\frac{3}{4}$ iſt B.	Therefore, C ($\frac{3}{4}$ is) B.
	(p. 360)
$\frac{3}{4}$ A ſind B.	Exactly $\frac{3}{4}$ of all A have predicate B
Alle C ſind A.	All C are A
Alle C $\frac{3}{4}$ ſind B.	All C ($\frac{3}{4}$ are) B.

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$\frac{3}{4}$ A ſind B.	Exactly $\frac{3}{4}$ of all A have predicate B
Etliche C ſind A.	Many C are A
Etliche C $\frac{3}{4}$ ſind B.	Many C ($\frac{3}{4}$ are) B.

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$\frac{2}{3}$ C sind A.	Exactly $\frac{2}{3}$ of all C are A
$\frac{2}{3}$ C $\frac{3}{4}$ sind B.	Exactly $\frac{2}{3}$ of all C ($\frac{3}{4}$ are) B.

The probability heuristics model (Chater & Oaksford, 1999; Oaksford & Chater, 2009)

Definitions of the basic sentences:

	Quantified statement	Prob. interpretation
(A)	All S are P	$p(P\|S) = 1$
(E)	No S is P	$p(P\|S) = 0$
(I)	Some S are P	$p(P\|S) > 0$
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(O)	Some S are not-P	$p(P\|S) < 1$
	Most S are P	$1 - \Delta < p(P\|S) < 1$
	Few S are P	$0 < p(P\|S) < \Delta$

... where Δ is small

The probability heuristics model (Chater & Oaksford, 1999, p. 201)

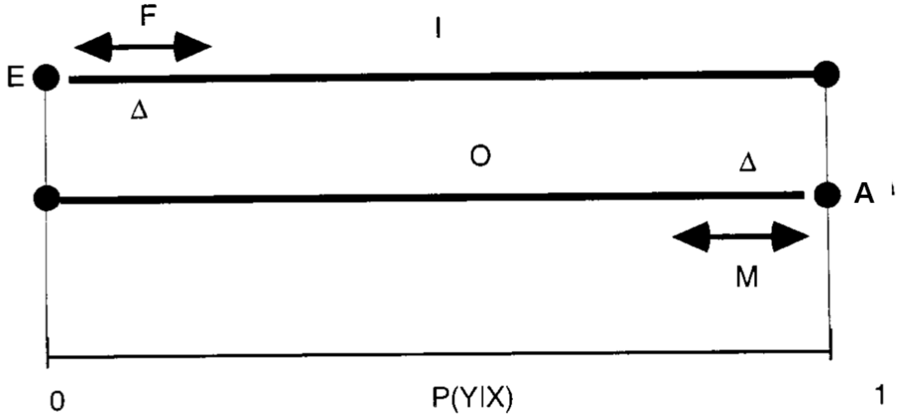

FIG. 2. The probabilistic semantics for the quantifers AMFIEO.

The probability heuristics model: Probabilistic syllogisms

- Assumption: Conditional independence between the end terms (i.e., S and P) given the middle term (i.e., M):

$$p(S \wedge P|M) = p(S|M)p(P|M)$$

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$$\begin{array}{rc} \text{(A)} & p(P|M) = 1 \\ \text{(A)} & p(M|S) = 1 \\ \text{(CI assumption)} & p(S \wedge P|M) = p(S|M)p(P|M) \\ \hline \text{(A)} & p(P|S) = 1 \end{array}$$

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Note, that we do not assume $p(S) > 0$ and $p(M) > 0$ in the coherence framework. Moreover, if $p(S|M) = 0$, then $p(S \wedge P|M) = 0$.

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Note, that we do not assume $p(S) > 0$ and $p(M) > 0$ in the coherence framework. Moreover, if $p(S|M) = 0$, then $p(S \wedge P|M) = 0$. Then, the premises are satisfied but $0 \leq p(P|S) \leq 1$ is coherent. Thus, Modus Barbara does not hold.

Towards Probabilistic Modus Barbara

$$\frac{\text{All } M \text{ are } P \qquad \text{All } S \text{ are } M}{\text{All } S \text{ are } P} \qquad \frac{p(P|M) = 1 \qquad p(M|S) = 1}{0 \leq p(P|S) \leq 1}$$

Towards Probabilistic Modus Barbara

All M are P	$p(P\|M) = 1$
All S are M	$p(M\|S) = 1$
All S are P	$0 \leq p(P\|S) \leq 1$

All M are P	$p(P\|M) = 1$
(Existential import: M	$p(M) > 0$)
All S are M	$p(M\|S) = 1$
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All S are M	$p(M\|S) = 1$
Existential import: S	$p(S) > 0$
All S are P	$p(P\|S) = 1$

If $p(S) = \gamma$ and $p(M|S) = 1$, then $\gamma \leq p(M) \leq 1$

Existential import: Different options

- Positive probability of the conditioning event, e.g.:

  $$\text{All } S \text{ are } P\text{: } p(S) > 0$$

- $p(S|M) > 0$ (and $p(M|P) > 0$) (Dubois, Godo, López de Màntaras, & Prade, 1993)

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- Replacing the first premise by a logical constraint, e.g.:

$$\begin{array}{c} \models (M \supset P) \\ p(M|S) = 1 \\ \hline p(P|S) = 1 \end{array}$$

- Strengthening the antecedent of the first premise, e.g.:

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$$\frac{\begin{array}{c}p(P|S \wedge M) = 1\\ p(M|S) = 1\end{array}}{p(P|S) = 1}$$

- Conditional event EI: Positive probability of the conditioning event, given the disjunction of all conditioning events (Gilio, Pfeifer, & Sanfilippo, 2016):

$$\frac{\begin{array}{c}p(P|M) = 1\\ p(M|S) = 1\\ p(S|S \vee M) > 0\end{array}}{p(P|S) = 1}$$

- $p(S|S \vee M) > 0$ neither implies $p(S) > 0$ nor $p(S|M) > 0$

Probabilistic Figure 1, conditional event EI

Premises		E.I.	Conclusion
$p(P\|M)$	$p(M\|S)$	$p(S\|S \vee M)$	$p(P\|S)$
x	y	t	$[z', z'']$
x	y	0	$[0,\ 1]$

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.9	1	1	[.9, .9]	
.9	1	.5	[.8, 1]	
.9	1	.2	[.5, 1]	
.9	1	.1	[0, 1]	

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1	$]0, 1]$	$t > 0$	$]0, 1]$	(Modus Darii)

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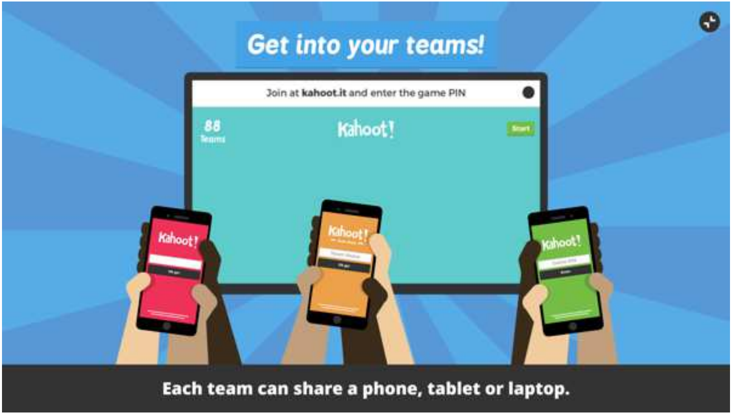
$.9$	1	$.1$	$[0,\ 1]$	
1	$]0,1]$	$t > 0$	$]0,1]$	(Modus Darii)

If $p(S|S \vee M) > 0$, then
$$z' = \max\left\{0, xy - \frac{(1-t)(1-x)}{t}\right\}$$
$$z'' = \min\left\{1, (1-x)(1-y) + \frac{x}{t}\right\}.$$

(Theorem 3 of Gilio, Pfeifer, and Sanfilippo (2015). *Transitive reasoning with imprecise probabilities*.)

Time for a quiz!



...and go to

kahoot.it

Syllogistic sentences as defaults (Gilio, Pfeifer, & Sanfilippo, 2016)

- Using our coherence interpretation, we also represent (A) by the following default:

$$S \mid\!\sim P \quad \text{(meaning: } p(P|S) = 1\text{)}$$

- ... its contradictory (O) by the negated default ($\neg(S \mid\!\sim P)$, short: $S \mid\!\not\sim P$):

$$S \mid\!\not\sim P \quad \text{(meaning: } p(P|S) < 1\text{)}$$

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Then, we interpret

- (E) by the default $S \mid\!\sim \neg P$ (meaning: $p(P|S) = 0$)
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Again, we do not presuppose that $p(S) > 0$!

Bridges to qualitative reasoning (e.g., Gilio, Pfeifer, & Sanfilippo, 2016)

The following versions of Weak Transitivity (Freund, Lehmann, & Morris, 1991) correspond to syllogisms and are theorems in our framework:

Modus Barbara:
$(B \mid\!\sim C, A \mid\!\sim B, A \vee B \mid\!\sim \neg A) \models_p A \mid\!\sim C.$

Modus Darii:
$(B \mid\!\sim C, A \mid\!\sim \neg B, A \vee B \not\mid\!\sim \neg A) \models_p A \not\mid\!\sim \neg C.$

Table of contents

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