University of Regensburg | Summer Term 2018

Quantum Theory of Condensed Matter I

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Mo. 08:00-10:00 c.t., PHY 5.0.21

Test Exam

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1. Quiz .... [5P]

(a)(1P) Why is the direct term ($q = 0$ term) of the Coulomb interaction $[(e^2/(2\epsilon_0 V \alpha^2))(\hat{N}^2 - \hat{N})]$ excluded in the Hamiltonian for the jellium model?

(b)(1P) Why is the RKKY[1]-interaction an indirect exchange interaction? Is it suitable for insulators?

(c)(1P) The energy of the ground state in the jellium-model was found to be $E_g/N_e = 2.21/r_s^2 - 0.916/r_s$. What are the origins of first and second summand in E_g? Considering the approximation made, what is the name of the missing contributions?

(d)(1P) Why is the ground state of the Heisenberg ferromagnet not unique? What is the degree of degeneracy if $S_{\text{tot}} = LS$?

(e)(1P) Sketch the typical band structure of a III-V zinc blende semiconductor arround the Γ-point (including both hole and electron states) and identify the respective bands.

2. Two Spins .... [5P]

Given are two localized spins which interact via

$$H = J\mathbf{S}_1 \cdot \mathbf{S}_2. \tag{1}$$

Calculate the spectrum of H. Depending on J, what is the ground state?

3. Tight Binding Model .... [5P]

Electrons on a 1D chain of N sites with a lattice constant a and periodic boundary conditions can be described in coordinate space by a tight binding Hamiltonian

$$H = -t \sum_{n=1}^{N} (c_i^\dagger c_{i+1} + \text{h.c.}) \tag{2}$$

(a)(3P) Calculate the spectrum $E(k)$.

(b)(2P) Calculate the effective mass m^*.

4. Correlation Functions .... [5P]

The one-particle correlation function in three dimensional coordinate space is given by

$$C_\sigma(\mathbf{x} - \mathbf{x}') = \langle \phi_0 | \Psi_\sigma^\dagger(\mathbf{x}) \Psi_\sigma(\mathbf{x}') | \phi_0 \rangle \tag{3}$$

where $|\phi_0\rangle$ is the ground state with Fermi wave vector k_F and Ψ are field operators, σ the spin quantum number.

(a)(2P) Write down G_σ using the operators $a_{\mathbf{k}\sigma}, a_{\mathbf{k}\sigma}^\dagger$ in Fourier space.

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[1] Rudermann-Kittel-Kasuya-Yosida

(b)(3P) Show that

$$C_\sigma(\mathbf{x}-\mathbf{x}') = \frac{3n}{2}\frac{\sin(rk_F) - rk_F\cos(rk_F)}{(rk_F)^3}, \quad r = |\mathbf{x}-\mathbf{x}'|, \quad n: \text{electron density} \tag{4}$$

using $\sum \to \int$.

5. Heisenberg Model .... [5P]

At some point of the derivation of the antiferromagnetic Heisenberg spectrum we encountered

$$H = JzS\sum_{\mathbf{k}}[\gamma_{\mathbf{k}}(c_{\mathbf{k}}d_{-\mathbf{k}} + d^\dagger_{\mathbf{k}}c^\dagger_{-\mathbf{k}}) + (c^\dagger_{\mathbf{k}}c_{\mathbf{k}} + d^\dagger_{\mathbf{k}}d_{\mathbf{k}})]. \tag{5}$$

Here, H is not written in the eigenbasis. Prove this statement. What is the name of the transformation which diagonalizes the Hamiltonian?

6. Magnons in a Ferromagnet .... [5P]

A one-magnon state is given by

$$|\mathbf{k}\rangle = \frac{1}{\hbar\sqrt{2SN}}S^-(\mathbf{k})\,|S\rangle\,, \quad \text{with} \quad S^\alpha(\mathbf{k}) = \sum_j e^{-i\mathbf{k}\cdot\mathbf{R}_j}S^\alpha_j, \quad \alpha = \{\pm, z\} \tag{6}$$

Using the commutation relations $[S^\alpha(\mathbf{k}), S^\beta(\mathbf{k}')]$, show that the magnon is a boson by evaluating $\langle\mathbf{k}|S^z_i|\mathbf{k}\rangle$.

Maximale Arbeitszeit: 120 Minuten
Zugelassene Hilfsmittel: handgeschriebene Formelsammlung (eine A4 Seite)
Notwendige Punktzahl zum bestehen: 10P