Quantum Theory of Condensed Matter I

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Sheet 8

The Kubo Formula for the Conductivity II .... [10]

We continue Ex.1, sheet 7. In the following we are going to rewrite the Kubo formula in a form which is often the starting point for treating the effect of disorder when calculating conductivity in wires.

(a)(5P) In the first step we assume to have the spectrum ϵ_n of the Hamiltonian $H = \sum_n \epsilon_n a_n^\dagger a_n$ with according eigenstates $|n\rangle$. Using the previous result from sheet 7, show that we can further rewrite $\sigma^{\beta\alpha}(\omega)$ as follows:

$$\sigma^{\beta\alpha}(\omega) = \mathrm{i}\hbar V \sum_{nm} \frac{f(\epsilon_n) - f(\epsilon_m)}{(\epsilon_m - \epsilon_n)(\epsilon_m - \epsilon_n + \hbar\omega + \mathrm{i}0^+)} \langle n|\tilde{j}^\alpha|m\rangle \langle m|\tilde{j}^\beta|n\rangle. \tag{1}$$

In this expression $\tilde{j}^\alpha = -ep^\alpha/(mV)$ with momentum p^α is an operator in the Schrödinger representation (in first quantization) and $f(\epsilon)$ is the Fermi-Dirac distribution function.

Hint: Remember, in second quantization the current operator is written as $j^\alpha(0) = \sum_{nm} \langle n| \tilde{j}^\alpha |m\rangle a_n^\dagger a_m$. You'll need the relation

$$\mathrm{Tr}[\rho a_m^\dagger a_n a_p^\dagger a_q] = \delta_{mq}\delta_{np} f(\epsilon_m)(1 - f(\epsilon_n)). \tag{2}$$

Furthermore, express $\theta(t - t') = (\mathrm{i}/2\pi) \int_{-\infty}^{\infty} dx \, \exp(-\mathrm{i}x(t - t'))/(x + \mathrm{i}0^+)$.

(b)(5P) The real part of the conductivity gives the dissipative contribution. Compute the real part of the longitudinal conductivity Eq. (1) in the DC limit, i.e.,

$$\mathrm{Re}[\sigma^{xx}(0)] = \frac{\pi e^2}{V m^2} \int_{-\infty}^{\infty} dE \left(-\frac{\partial f(E)}{\partial E} \right) \mathrm{Tr}[p^x \delta(E - H) p^x \delta(E - H)]. \tag{3}$$

2. Two Coupled Spins .... [10P]

Once again we use the Heisenberg model. However, this time we only consider two spin 1/2 particles, $S_1 = S_2 = 1/2$, in an external magnetic field B:

$$H = -J(S_1^+ S_2^- + S_1^- S_2^+ + 2S_1^z S_2^z) - \frac{g_J \mu_B B}{\hbar}(S_1^z + S_2^z). \tag{4}$$

A complete system of equations of motion is given by using the following (retarded or advanced) Green's functions:

$$G_{11}(t, t') = \langle\langle S_1^-(t); S_1^+(t') \rangle\rangle_{\epsilon=+} \tag{5}$$
$$G_{21}(t, t') = \langle\langle S_2^-(t); S_1^+(t') \rangle\rangle_{\epsilon=+} \tag{6}$$
$$\Gamma_{12}(t, t') = \langle\langle S_1^z(t) S_2^-(t); S_1^+(t') \rangle\rangle_{\epsilon=+} \tag{7}$$
$$\Gamma_{21}(t, t') = \langle\langle S_2^z(t) S_1^-(t); S_1^+(t') \rangle\rangle_{\epsilon=+} \tag{8}$$

Show that the Green's functions can be written as

$$G_{11}(E) = \sum_{i=1}^{3} \frac{a_i}{E - E_i}, \quad G_{21}(E) = \sum_{i=1}^{3} \frac{b_i}{E - E_i} \tag{9}$$

and

$$\Gamma_{12}(E) - \Gamma_{21}(E) = \sum_{i=2}^{3} \frac{c_i}{E - E_i}. \tag{10}$$

Determine the coefficients a_i, b_i and c_i, which are functions of $\langle S_1^z \rangle = \langle S_2^z \rangle \equiv \langle S^z \rangle$ and $\rho_{12} \equiv \langle S_1^+ S_2^- \rangle + 2\langle S_1^z S_2^z \rangle$, and the poles E_i.