Quantum Theory of Condensed Matter I

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Sheet 0

1. Two Spins in a Time Dependent Potential (8 points)

Consider a system made up of two $s = \hbar/2$ spins. The spin interaction is switched on at $t = 0$, the Hamiltonian describing it is given by

$$H = \frac{4\Delta}{\hbar^2}\mathbf{S}_1 \cdot \mathbf{S}_2. \tag{1}$$

For $t < 0$ the system is in the state $|+-\rangle$ (the Hamiltonian is only a constant which can be set to zero), where $|\pm\pm\rangle \equiv |S_z, \pm\rangle \otimes |S_z, \pm\rangle$. Find, as a function of time, the probability for the system being found in each of the following states: $|++\rangle, |+-\rangle, |-+\rangle, |--\rangle$. *Hint: Rewrite H using the ladder operators. What is the ground state depending on Δ?*

2. Conservation of Particle Number (4 points)

Show that the system described by the Hamiltonian H

$$H = \int d^3\mathrm{k}\, \frac{\hbar^2 k^2}{2m} a^\dagger_{\mathbf{k}} a_{\mathbf{k}} + \frac{1}{2} \iiint d^3\mathrm{k}\, d^3p\, d^3q\, V(\mathbf{q}) a^\dagger_{\mathbf{k}+\mathbf{q}} a^\dagger_{\mathbf{p}-\mathbf{q}} a_{\mathbf{p}} a_{\mathbf{k}} \tag{2}$$

conserves the particle number N of eigenstates of this system. Does it depend on whether the operators are fermionic or bosonic?

3. Two Level System (8 points)

We assume a two-level system of $N = 0, \ldots, 4$ electrons with spin $\sigma =\uparrow, \downarrow$ (in the eigenbasis of σ_z) described by the following Hamiltonian

$$H = \sum_\sigma (E_1 c^\dagger_{1\sigma} c_{1\sigma} + E_2 c^\dagger_{2\sigma} c_{2\sigma} + V(c^\dagger_{1\sigma} c_{2\sigma} + c^\dagger_{2\sigma} c_{1\sigma})). \tag{3}$$

a) Derive the eigenvalue equation for arbitrary N using the Fock-states $|N, l\rangle \equiv |N, n_{1\uparrow} n_{1\downarrow} n_{2\uparrow} n_{2\downarrow}\rangle$, where l numbers the possible Fock-states for a fixed N. *Hint:* Explain and use $\langle N, l\, |H|\, N', l'\rangle \sim \delta_{NN'}$.

b) Calculate the eigenvalues for $N = 0, \ldots, 4$. *Hint:* In the case of $N = 2$ we have 6 Fock-states. However: Two of them are already eigenstates which reduces the problem again to a 4×4 secular determinant!