# General Relativity and Cosmology

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**Mo.** H34 12pm c.t. & **Wed.** PHY 9.2.01, 1pm c.t.
**Thu.** 1pm c.t., PHY 9.1.10

## Sheet 8

### 1) Planar Motion .......... [7P]

(a)(2P) Prove that all orbits in the Schwarzschild metric are planar.

(b)(5P) Show that all these orbits are stable, i.e., a perturbation $\delta\theta$ of the orbit with $\theta = \pi/2$, $L = p_\varphi =: L_0$ is restricted by an oscillation about $\theta = \pi/2$ up to second order in $\delta\theta$.

### 2) Falling into a Black Hole .......... [10P]

In the following, assume Schwarzschild metric.

(a)(5P) A spaceship is falling into a black hole of mass $M$. It has an impact parameter of $r_0 = 4r_s$ and an asymptotic speed of $\lim_{r\to\infty} \dot{r} \equiv v_\infty = c/\sqrt{2}$. Is the spaceship going to fall into the center of the black hole?

(b)(5P) Now assume the angular momentum of the spaceship to be zero, the ship is free falling into the center from position $r(t=0) = 3r_s$.

- How long does it take to reach the center according to proper time? Sketch the function $r(\tau)$.
- Solve the equation of motion of the spaceship for $r(t)$ assuming $r \approx r_s$. How does $r(t)$ look like in case of a photon?

### 3) Orbiting Photon .......... [10P]

In the following, use the Schwarzschild metric to describe the space-time outside a spherical mass $M$.

(a)(2P) A photon describes a circular orbit around $M$. What is the radius $r_c$ of this motion?

(b)(4P) What is the period of the photon orbit measured by an observer at this radius?

(c)(4P) The observer $A$ at $r_c$ sends a light signal to an observer $B$ at $r \gg GM/c^2$ each time the photon passes him. Which interval for the incoming signals is $B$ measuring?

### 4) $\pi$ .......... [7P]

Assume the earth orbit around the sun to be a circle with radius $r_0$. Let the gravitation field of the sun be described by the inner and outer Schwarzschild metric. Assume further the sun to have a homogeneous density. Calculate the ratio of earth orbital's circumference $C$ to the diameter $D$.

### 5*) Gravitational Lensing

Lensing occurs when the path of light from a source travels near a massive object. Write a program (either in C/C++ or Mathematica or Pyhton or . . . ) which calculates the image created by a point mass lens (possible parameters: distance to lens $\sim 10^9$ ly, mass of lens $M \sim 10^{12} M_\odot$, radius of circular image $\sim 1$").

- Start with a point source which is collinear with the point mass lens and the observer. Reproduce an Einstein ring.

- Assume $S$ to be the surface with a surface normal parallel to the line through lens and observer. Calculate the apparent position of the point light source for the observer when the source is set at an arbitrary point in $S$.
- Calculate the image of a disk created by the point mass lens for a collinear and non-collinear position.

*Supplementary material: arXiv:astro-ph/9606001v2, Wolfram Demonstration Projects.*