General Relativity and Cosmology

Prof. John Schliemann   **Mo.** H34 12pm c.t. & **Wed.** PHY 9.2.01, 1pm c.t.
Dr. Paul Wenk   **Thu.** 1pm c.t., PHY 9.1.10

Sheet 1

1) Muons .... [5P]

(a)(2P) A muon at rest lives 2.2×10^{-6} s and has a mass of $105.7 \, \text{MeV}/c^2$. Which energy must a muon have to reach Earth's surface if it is produced at $10 \, \text{km}$ above the surface [nearly 90% of all muons are produced at this height]?

(b)(3P) Approximate the Earth's magnetic field in zeroth order to be $30 \, \mu\text{T}$ in the direction of its axis. Calculate the deflection of a muon of the energy given by (a) due to the field. The muon is normally incident at the equator.

2) Light Beam .... [5P]

A light beam is propagating through a block of glass with an index of refraction n. Assume further, the glass is moving at a constant velocity v in the laboratory in the same direction as the beam. Calculate the velocity of light in the block as measured by an observer in the laboratory.

3) Geometry in Minkowski diagrams...................................... [7P]

Let some inertial frame (primed coordinates) move with velocity v relatively to another inertial frame (unprimed coordinates). Show the following:

(a)(3P) Assume that light is emitted at (t_0, x_0), reflected by a mirror at x_m and measured at (t_2, x_0), see Fig.. A clock at the position of the mirror which shows the time t_1 is said to be synchronous with a clock at the position x_0, if $t_1 = t_0 + (t_2 - t_0)/2$ holds for every t_0. Use this measuring instruction to show that the slope of the simultaneity line of the primed system is v/c.

(b)(3P) The tangent to the hyperbola $c^2 t^2 - x^2 = c^2 t_0^2$ in the intersection point with the line $x' = 0$ is parallel to the line $ct' = 0$, i.e. a line of constant ct'. What is the value of ct' and where does the tangent intersect with the line $x = 0$?

(c)(1P) Analogous to (a), do the calculation for the hyperbola $c^2 t^2 - x^2 = -l_0^2$ and line $ct' = 0$.
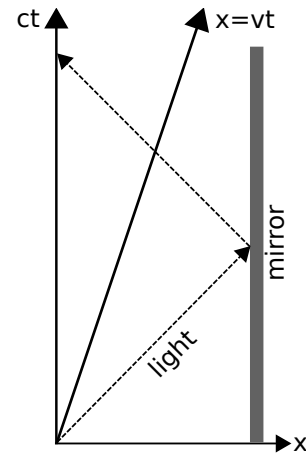
4) Twin Paradox .... [5P]

A spaceship sets off from earth whose rest frame is assumed to be an inertial frame. For the first 5 years of their proper time the astronauts feel an acceleration $a = g = 9.81 \text{m/s}^2$, the next five years the spaceship decelerates with $a = -g$. After coming to rest, the spaceship returns to earth the inverse way. What time has elapsed for an earth observer upon the ship's return, and what distance has the ship traveled (in the earth rest frame)?