Control of Spin Helix Symmetry in Semiconductor Quantum Wells by Crystal Orientation

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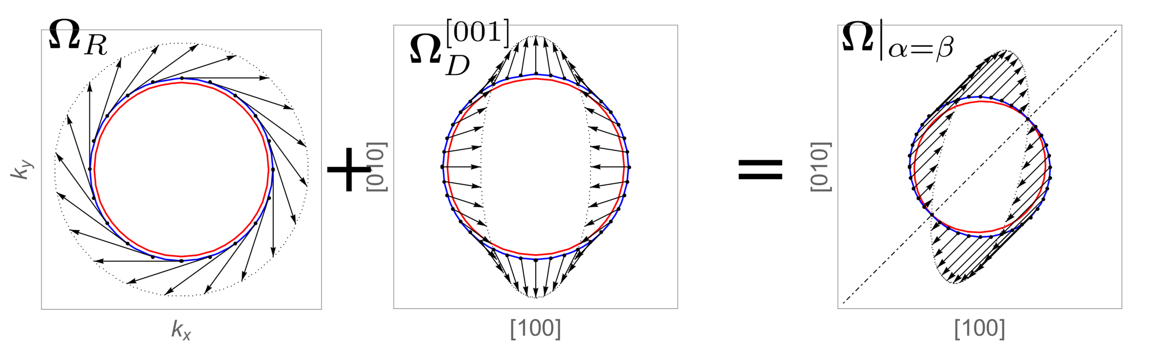
Motivation

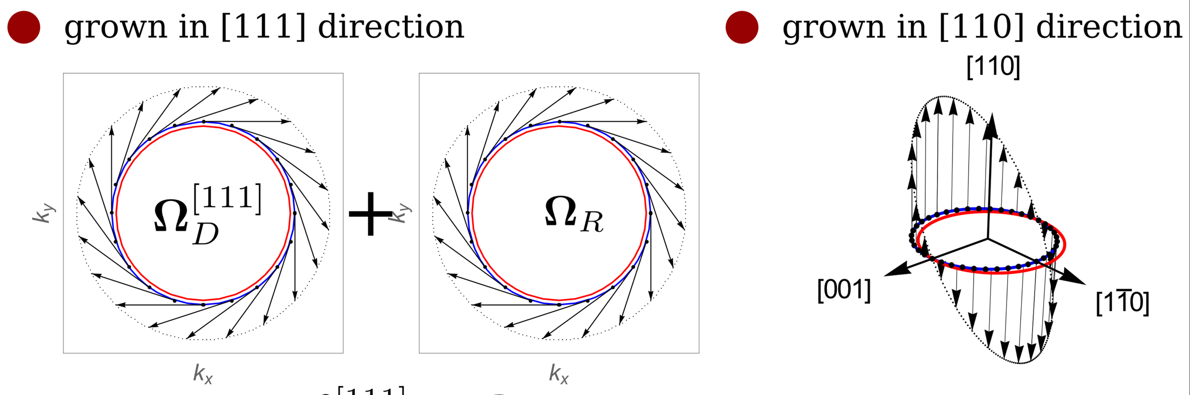
We investigate the possibility of spin-preserving symmetries due to the interplay of **Rashba (R)** and **Dresselhaus (D)** spin-orbit coupling in **n-doped zinc-blende** semiconductor quantum wells.
There are special cases known in 2DEGs where the effective magnetic field (SOF) $\boldsymbol{\Omega}$ due to SOC is collinear:

$$\mathcal{H}_{\rm SO} = (\boldsymbol{\Omega}_{\rm R} + \boldsymbol{\Omega}_{\rm D})\cdot\boldsymbol{\sigma}, \qquad \|\boldsymbol{\Omega}_{\rm R}\| \propto \alpha \propto V_{\rm G}, \|\boldsymbol{\Omega}_{\rm D}\| \propto \beta$$

- grown in [001] direction



- grown in [111] direction
- grown in [110] direction

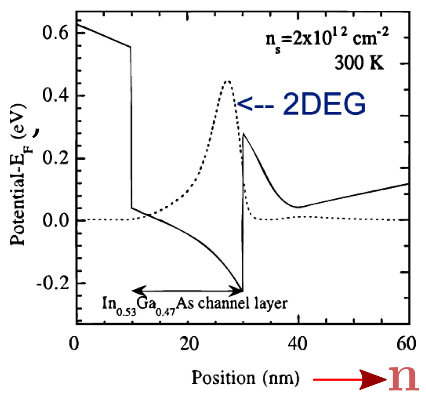


$\alpha = -\beta^{[111]} \Rightarrow \boldsymbol{\Omega} = 0$ — Collinear for $\alpha = 0$

- Is it possible to find conserved spin operators for other growth directions (even with cubic D-SOC)? What are the sufficient and necessary conditions?
- Are the long-lived/persistent spin states of helical nature?

System

The analyzed system is a 2DEG confined by an electric field $\boldsymbol{\mathcal{E}} = \mathcal{E}_0\mathbf{n}$, with the normal $\mathbf{n} = \{n_x, n_y, n_z\}$.



The model for the lowest subband is given by

$$\mathcal{H} = \frac{\hbar^2 k^2}{2m} + \boldsymbol{\Omega}\cdot\boldsymbol{\sigma}$$

with the SO field $\boldsymbol{\Omega} = \boldsymbol{\Omega}_{\rm R} + \boldsymbol{\Omega}_{\rm D}^{(1)} + \boldsymbol{\Omega}_{\rm D}^{(3)}$

due to **Rashba** $\boldsymbol{\Omega}_{\rm R} = \alpha\,(\mathbf{k}\times\mathbf{n})$, $\gamma_R\mathcal{E}_0$

and **Dresselhaus** SOC

$$\boldsymbol{\Omega}_{\rm D} = \boldsymbol{\Omega}_{\rm D}^{(1)} + \boldsymbol{\Omega}_{\rm D}^{(3)} = \gamma_{\rm D}\boldsymbol{\nu} \quad \text{with}$$

$$\nu_x = \left(\frac{\pi}{a}\right)^2\left[2n_x(n_y k_y - n_z k_z) + k_x(n_y^2 - n_z^2)\right] + k_x\left(k_y^2 - k_z^2\right)$$

and c.p. for y, z components

and the boundary condition $\mathbf{k}\cdot\mathbf{n} = 0$.

Assumption: Contribution coming from 3rd order angular harmonic $\boldsymbol{\Omega}_{\rm D}^{(3)}$ can be neglected. We are left with

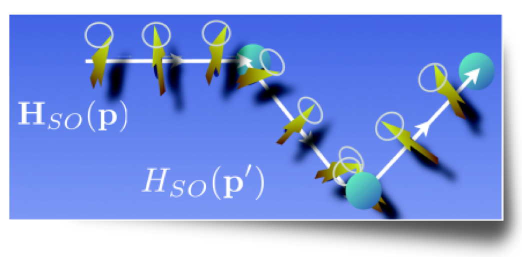
$$\boldsymbol{\Omega}_{\rm D}^{(1)} = \beta^{(1)}\boldsymbol{\kappa}$$

$$\beta^{(1)} = \gamma_{\rm D}\left[(\pi/a)^2 - k^2/4\right]$$

shift does not depend on $\mathbf{n}$

Now we include also non-magnetic impurities into the system.

randomization of spin



Spin Diffusion

Assumptions:

(1) Uncorrelated disorder: $\langle V\rangle = 0, \langle V(\mathbf{x})V(\mathbf{x}')\rangle \propto \delta(\mathbf{x}-\mathbf{x}')$

(2) weak disorder: $k_F l_e \gg 1$

The dominant mechanism for spin relaxation in systems with broken bulk inversion symmetry (III-V compounds) and $\tau_e\Omega_{\rm SO} \lesssim 1$

D'yakonov Perel' spin relaxation

with the according diffusion eq. for the spin density

$$0 = \left(D_{\rm e}q^2 - i\omega + 1/\hat{\tau}_{\rm s}^{(0)}\right)\mathbf{s} + \frac{4i\tau_{\rm e}}{m}\langle(\mathbf{q}\cdot\mathbf{k})\,\boldsymbol{\Omega}\rangle\times\mathbf{s}$$

with the spin relaxation tensor

$$\left(1/\hat{\tau}_{\rm s}^{(0)}\right)_{ij} = 4\tau_{\rm e}/\hbar^2\left(\langle\boldsymbol{\Omega}^2\rangle\,\delta_{ij} - \langle\Omega_i\Omega_j\rangle\right)$$

$$0 = (D_e\Lambda_{\rm sd}(\mathbf{q}) - i\omega)\,\mathbf{s}$$

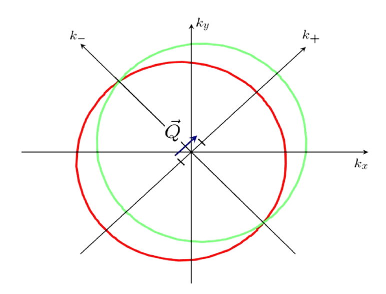
$D_{\rm e} = v_{\rm F}^2\tau_{\rm e}/2$

averaging over all in-plane directions of k

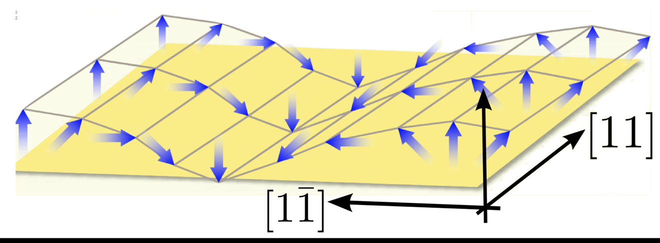
Known: $\Lambda_{\rm sd}(\mathbf{q})$ for pure Rashba or Dresselhaus SOC.

Here we extend the analysis:

(1) Calculation of $\Lambda_{\rm sd}(\mathbf{q})$ for an arbitrary growth direction.
(2) Finding eigenvalues λ_i of $\Lambda_{\rm sd}(\mathbf{q})$
(3) Identifying longest spin lifetimes.



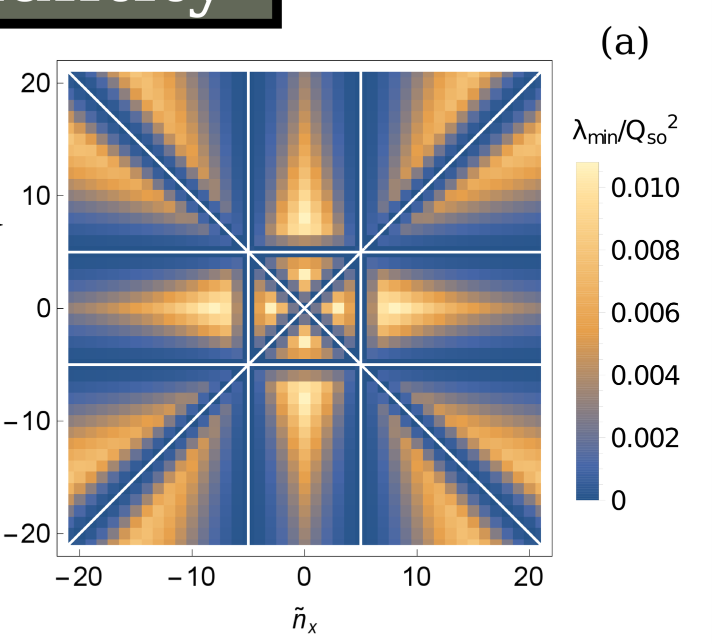
For $\mathbf{q}_{\rm min} \neq 0 \Rightarrow$ persistent spin HELIX

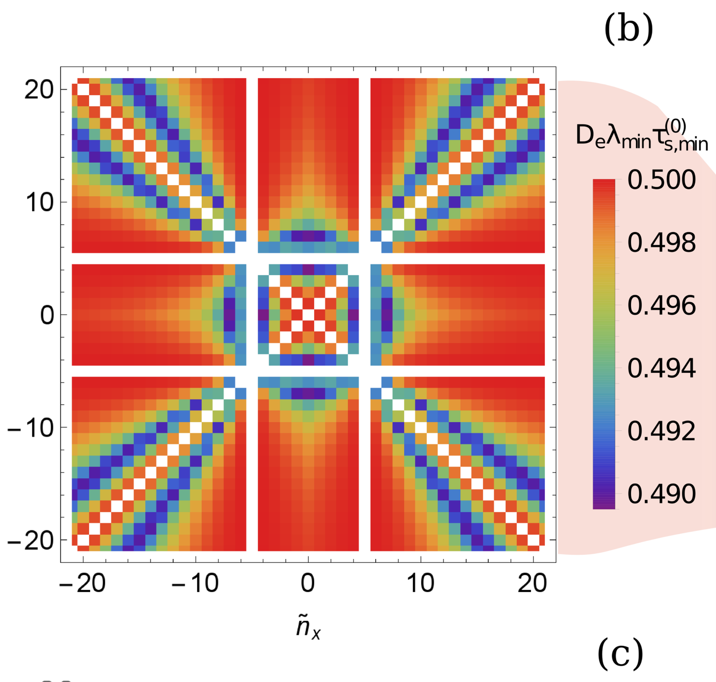


Conserved Spin Quantity

On the RHS (a) the global minimum $\lambda_{\rm min}$ of the spectrum of the spin diffusion operator $\Lambda_{\rm sd}$ for the optimal ratio of Rashba and Dresselhaus coefficients $\alpha/\beta^{(1)}$ is plotted assuming $\tilde{\mathbf{n}} = \{\tilde{n}_x, \tilde{n}_y, 5\}$

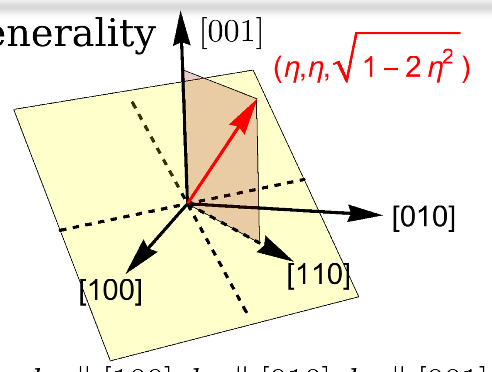
- **If two Miller indices are equal in modulus**
  - $\lambda_{\rm min} \equiv \lambda(\mathbf{q}_{\rm min}) \sim 1/\tau_s = 0$
  - $\mathbf{q}_{\rm min} \neq 0 \Rightarrow$ persistent spin helix Fig.(c)
- else $\lambda_{\rm min}(\mathbf{q}\neq 0)/\lambda_{\rm min}(0) \approx 1/2$, Fig.(b)





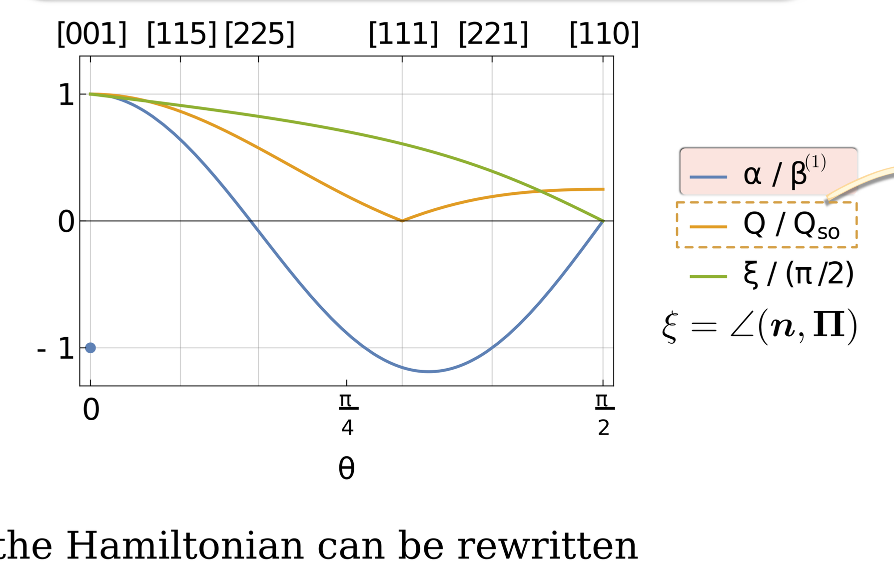
Using, without loss of generality

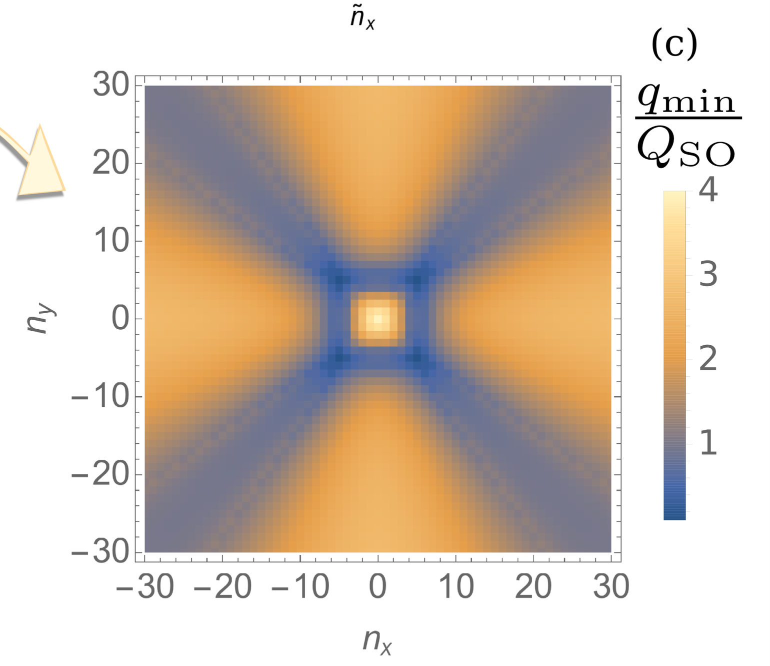
$n_i > 0$
$n_x = n_y = \eta$
$n_z = \sqrt{1-2\eta^2}$
$\eta \in [0, 1/\sqrt{2}]$



$k_x \parallel [100], k_y \parallel [010], k_z \parallel [001]$

$$\alpha/\beta^{(1)} = \Gamma_0 := (1-9\eta^2)\sqrt{1-2\eta^2}$$





the Hamiltonian can be rewritten

$$\mathcal{H} = \frac{\hbar^2}{2m}\left(k^2 + (\mathbf{k}\cdot\mathbf{Q})\,\Sigma\right)$$

with $\Sigma = \left(\sigma_x + \sigma_y + \frac{3\eta\sqrt{1-2\eta^2}}{3\eta^2-1}\sigma_z\right)/N$ and $\mathbf{Q} = \frac{Q_0}{\sqrt{2}}(-1, 1, 0)$

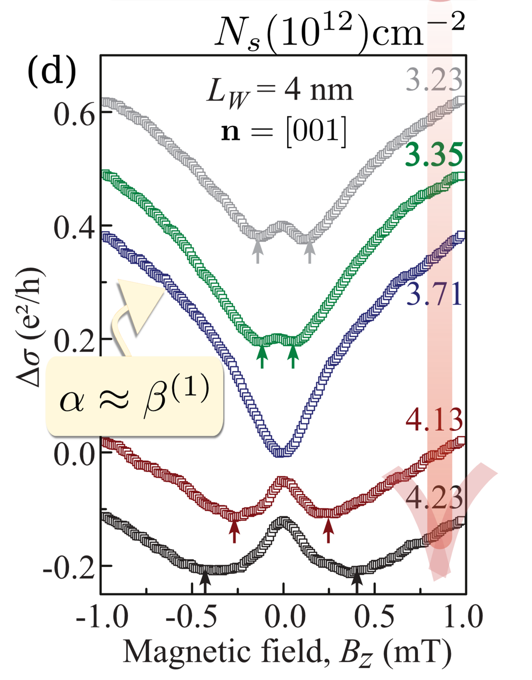
conserved spin quantity $[\mathcal{H}, \Sigma] = 0$

$$N = \sqrt{2-3\eta^2}/|1-3\eta^2|$$
$$Q_0 = |1-3\eta^2|\sqrt{1-3\eta^2/2}\,Q_{\rm so}$$
$$Q_{\rm so} = 4m\beta^{(1)}/\hbar^2$$

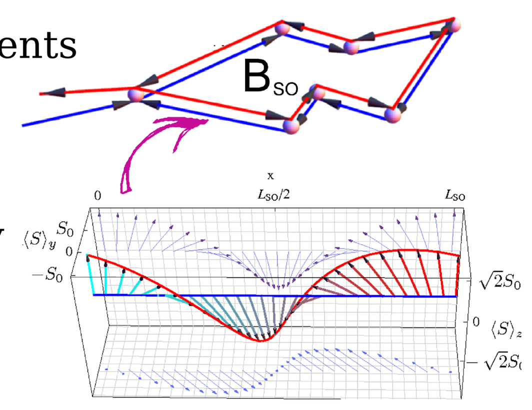
- for $\mathbf{n} \parallel [111] \vee [110]$ persistent states even with including $\boldsymbol{\Omega}_{\rm D}^{(3)}$
- $\boldsymbol{\Sigma} \equiv \boldsymbol{\Pi}\cdot\boldsymbol{\sigma}$ $\rightarrow \boldsymbol{\Pi} \perp \boldsymbol{Q}$

Measurement: Weak (Anti)Localization

$$\beta^{(1)} \sim \left(\frac{\pi}{a}\right)^2 - \frac{\pi}{2}N_s$$



Magnetoconductivity (MC) measurements allow for extracting SOC parameters. One characteristic quantity: Minimum in the magnetic field B of the correction to static conductivity



$$\Delta\sigma = \frac{2e^2}{h}\int_{\mathcal{Q}<\sqrt{c_e}}\frac{d^2\mathcal{Q}}{(2\pi)^2}\left(\frac{1}{\mathcal{Q}^2+c_\phi+c_{\rm B}} - \sum_{j\in\{\pm1,0\}}\frac{1}{\lambda_j(\mathcal{Q})/Q_{\rm so}^2+c_\phi+c_{\rm B}}\right),$$

$$c_i = 1/(D_e Q_{\rm so}^2\tau_i),\ i\in\{\phi, {\rm e}, {\rm B}\} \qquad 1/\tau_{\rm B} = 2D_{\rm e}e|B|/\hbar$$

We used the equivalence of the Cooperon spectrum and $\Lambda_{\rm sd}$

If the MC is measured near the SU(2) symmetry <=> $\alpha/\beta^{(1)} \mapsto \Gamma_0 + \epsilon$

approximate the spectrum of the triplet sector of the Cooperon by three parabolas of the form

$$\lambda_j/Q_{\rm so}^2 = \mathcal{Q}_+^2 + (\mathcal{Q}_- + j\xi)^2 + \Delta_{|j|} \qquad j\in\{0,\pm1\}$$

$$\Delta_0 \approx 2\Delta_1 \approx \epsilon^2/4$$

The minima of $\lambda_\pm$ are shifted to finite in-plane wave vectors

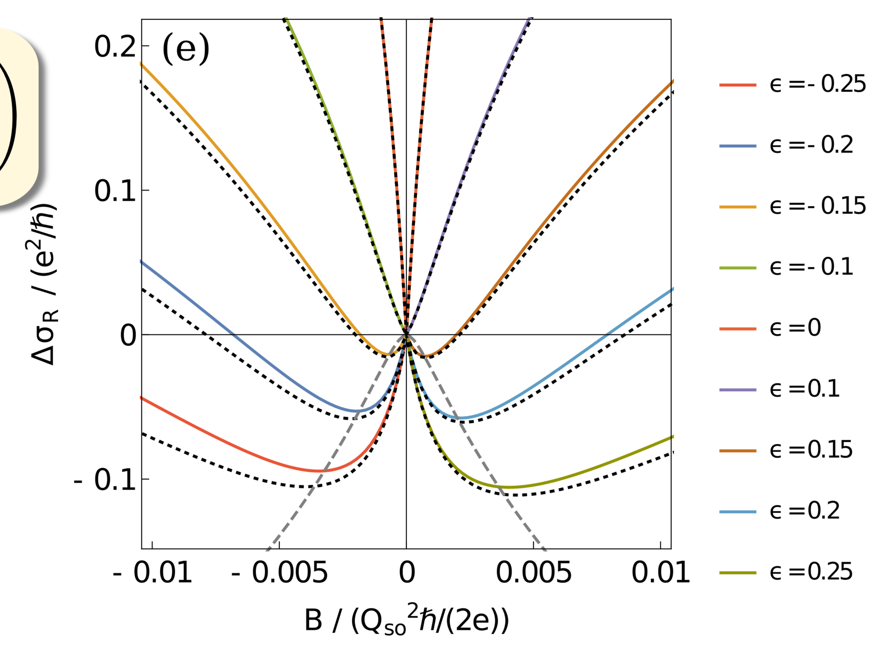
$$\mathcal{Q}_- = \pm\xi\ ,\ \xi^2 \approx Q_0^2/Q_{\rm so}^2 + \Delta_0 + \epsilon(1-3\eta^2)\sqrt{1-2\eta^2}$$

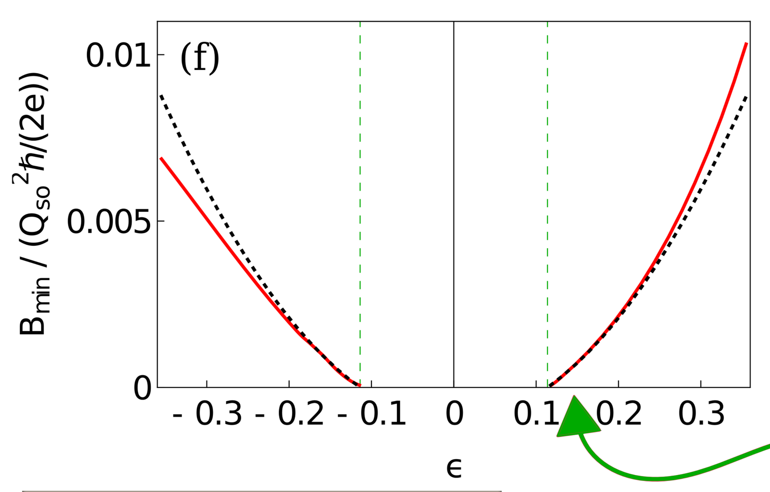
which are oriented along $[\bar{1}10]$, representing the long-lived helical spin states.

$$\Delta\sigma \approx \frac{e^2}{2\pi h}\ln\left(\frac{4\Upsilon_{100}\Upsilon_{010}\Upsilon_{001}^2}{\Upsilon_{000}\Upsilon_{110}\left(\Upsilon_{101}-\xi^2+\sqrt{\upsilon}\right)^2}\right)$$

$$\upsilon = \Upsilon_{101}^2 + 2\Upsilon_{-101}\xi^2 + \xi^4$$
$$\Upsilon_{jkl} = c_\phi + c_{\rm B} + jc_e + k\Delta_0 + l\Delta_1$$

Now we can extract $B_{\rm min}$ where the minimum in the WAL can be found:





$$B_{\rm min} \approx \frac{(\sqrt{5}-1)m^2\tilde{\alpha}^2}{2e\hbar^3} - \frac{\hbar}{2eD_{\rm e}\tau_\phi} \qquad \tilde{\alpha} = \beta^{(1)}\epsilon$$

WL/WAL transition: $\tilde{\alpha}^2 \approx (1+\sqrt{5})\hbar^4/(4D_e m^2\tau_\phi)$

References

Poster results: arXiv:1606.08774 (2016)

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