Direction Dependence of Spin Relaxation and Diffusive-Ballistic Crossover

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Abstract

The dependence of spin relaxation on the direction of the quantum wire under Rashba and Dresselhaus (linear and cubic) spin orbit coupling (SOC) is studied using the Cooperon equation. Comprising the dimensional reduction of the wire in the diffusive regime, the lowest spin relaxation and dephasing rates for (001) and (110) systems are found. The analysis of spin relaxation is then extended to non-diffusive wires where it is shown that, in contrast to the theory of dimensional crossover from weak localization to weak antilocalization in diffusive wires, the relaxation due to cubic Dresselhaus spin orbit coupling is reduced and the linear part shifted with the number of transverse channels.[1–3]
We set $\hbar \equiv 1$.

Cooperon and Spin Diffusion

The weak localization correction to the conductivity is given by

$$\Delta\sigma = -\frac{2e^2}{2\pi}\frac{D_e}{\text{Vol.}}\sum_{\mathbf{Q}}\sum_{\alpha,\beta=\pm}\mathcal{C}_{\alpha\beta\beta\alpha,\omega=0}(\mathbf{Q}), \qquad (1)$$

where $\alpha,\beta=\pm$ are the spin indices, and the Cooperon propagator $\hat{\mathcal{C}}$ is for $\epsilon_F\tau \gg 1$ (ϵ_F, Fermi energy), given by

$$\hat{\mathcal{C}}_{\omega=E-E'}(\mathbf{Q}=\mathbf{p}+\mathbf{p}') = \tau\left(1-\sum_{\mathbf{q}}\overset{E,\mathbf{p}+\mathbf{q}}{\underset{E',\mathbf{p}'-\mathbf{q}}{\rightleftarrows}}\right)^{-1}. \qquad (2)$$

Expanding $\hat{\mathcal{C}}$ to lowest order in the generalized momentum $\mathbf{Q}$ leads to

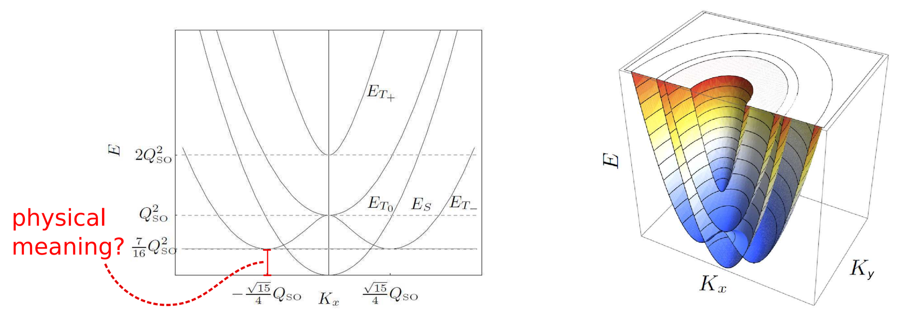
$$\hat{\mathcal{C}}_{\omega=0}(\mathbf{Q}) \equiv \hat{\mathcal{C}}(\mathbf{Q}) = \left(D_e(\mathbf{Q}+2e\mathbf{A}+2e\mathbf{A_S})^2+H_\gamma\right)^{-1} \qquad (3)$$

e.g. in GaAs (001), with the Rashba parameter α_2 and the shifted linear Dresselhaus coupling $\tilde{\alpha}_1=\alpha_1-m_e\gamma_D E_F/2$,

$$\mathbf{A_S} = \frac{m_e}{e}\hat{\alpha}.\mathbf{S} = \frac{m_e}{e}\begin{pmatrix}-\tilde{\alpha}_1 & -\alpha_2 & 0\\ \alpha_2 & \tilde{\alpha}_1 & 0\end{pmatrix}.\mathbf{S}, \qquad (4)$$

$$H_\gamma = (m_e^2\gamma\epsilon_F)^2(S_x^2+S_y^2). \qquad (5)$$

2D spectrum of $H_c := \frac{\hat{C}^{-1}}{D_e}$ splits in gapless singlet and triplet modes



The local corrections given by $\hat{C}$ can be related to spin relaxation (requiring time reversal symmetry)

$$H_c = U_{\text{CD}}H_{\text{SD}}U_{\text{CD}}^\dagger$$

with the spin diffusion equation for $(v_F \mid \nabla_{\mathbf{r}}\mathbf{s}\mid) \ll 1/\tau$

$$0 = \partial_t\mathbf{s} + \underbrace{\frac{1}{\hat{\tau}_s}\mathbf{s}}_{\text{spin relax.}} - D_e\nabla^2\mathbf{s} + \underbrace{\gamma(\mathbf{B}-2\tau\langle(\nabla\mathbf{v}_F)\mathbf{B}_{\text{SO}}(\mathbf{k})\rangle)\times\mathbf{s}}_{\text{spin precession}}$$

$$0 = \partial_t\mathbf{s} + D_e H_{\text{SD}}\mathbf{s} \qquad (6)$$

with the D'yakonov-Perel' Spin Relaxation Tensor, $\mathbf{B}=0$

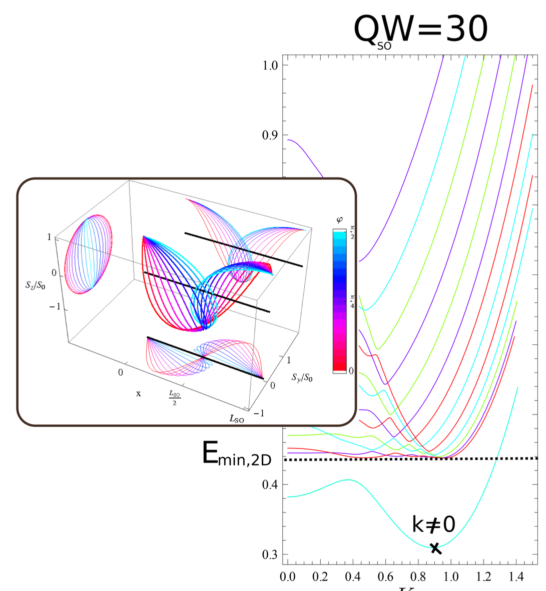
$$\frac{1}{\tau_{sij}} = \tau\gamma^2\left(\langle\mathbf{B}_{\text{SO}}(\mathbf{k})^2\rangle\delta_{ij} - \langle B_{\text{SO}}(\mathbf{k})_i B_{\text{SO}}(\mathbf{k})_j\rangle\right).$$

Spin Relaxation Anisotropy in the (001) System

Consider specular scattering from spin-conserving boundaries:

$$\mathbf{n}\cdot\mathbf{j}_{s_i}|_{\pm\frac{W}{2}} = 0,$$

$$\mathbf{j}_{s_i} = -\tau\langle\mathbf{v}_F\left(\mathbf{B}_{\text{SO}}(\mathbf{k})\times\mathbf{s}\right)_i\rangle - D_e\nabla s_i.$$



Using U_{CD} we find the BC for the Cooperon and simplify them by applying a second transformation U_A to simple Neumann BC:

$$U_A\left(\frac{\tau}{D_e}\mathbf{n}\cdot\langle\mathbf{v}_F((\mathbf{B}_{\text{SO}}(\mathbf{k})\cdot\mathbf{S}))\rangle - \mathrm{i}\partial_{\mathbf{n}}\right)U_A^\dagger U_A C U_A^\dagger|_{\text{Surface}} = 0$$

$$U_A\left(\mathbf{n}\cdot 2e\mathbf{A_S} - \mathrm{i}\partial_{\mathbf{n}}\right)U_A^\dagger\tilde{C}|_{\text{Surface}} = 0,$$

$$-\mathrm{i}\partial_{\mathbf{n}}\tilde{C}|_{\text{Surface}} = 0$$

Solving the Cooperon Hamiltonian H_c with BC for different directions $\mathbf{n}$, $(\theta=0:\mathbf{n}=\hat{e}_y)$, in the (001) plane gives the minima

$$E_{1/2,min} = \frac{3}{2}\frac{q_{s3}^2}{2} + \frac{\left(q_{sm}^2-\frac{q_{s3}^2}{2}\right)\left(\alpha_{x1}^2+\alpha_{x2}^2-q_2^2\right)^2}{24q_{sm}^2}W^2, \qquad (7)$$

$$E_{3,min} = \frac{q_{s3}^2}{2} + \frac{\left(\frac{q_{s3}^2}{2}+q_{sm}^2\right)\left(\alpha_{x1}^2+\alpha_{x2}^2-q_2^2\right)^2}{12q_{sm}^2}W^2 \qquad (8)$$

where we set $q_{sm}=\sqrt{(\alpha_{x2}-q_2)^2+\alpha_{x1}^2}$ and

$$\alpha_{x1} = \frac{1}{2}m_e\gamma_D\cos(2\theta)((m_e v)^2-4\langle k_z^2\rangle),$$
$$\alpha_{x2} = -\frac{1}{2}m_e\gamma_D\sin(2\theta)((m_e v)^2-4\langle k_z^2\rangle),$$
$$q_2 = 2m_e\alpha_2,\ \ q_{s3}^2/2 = (m_e^2\epsilon_F\gamma_D)^2.$$

In the Fig. above, the spectrum for $\theta=0$ at $q_2W=30$ is plotted. The absolute minimum at finite wave vectors K_x leads to **long persisting spin helices** as shown in the inset.
For a general θ we can deduce about the minimal spin-relaxation rate that Eq. (7) is independent of the width W if $\alpha_{x1}(\theta=0)=-q_2$ and/or the direction of the wire is pointing in

$$\theta = \frac{1}{2}\arcsin\left(\frac{2\langle k_z^2\rangle(m_e\gamma_D)^2((m_e v)^2-2\langle k_z^2\rangle)-q_2^2}{(m_e^3v^2\gamma_D-4\langle k_z^2\rangle m_e\gamma_D)\,q_2}\right). \qquad (9)$$

Analyzing the prefactor of W^2 in the Eq. (8) gives the optimal angle as plotted in Fig. 1.

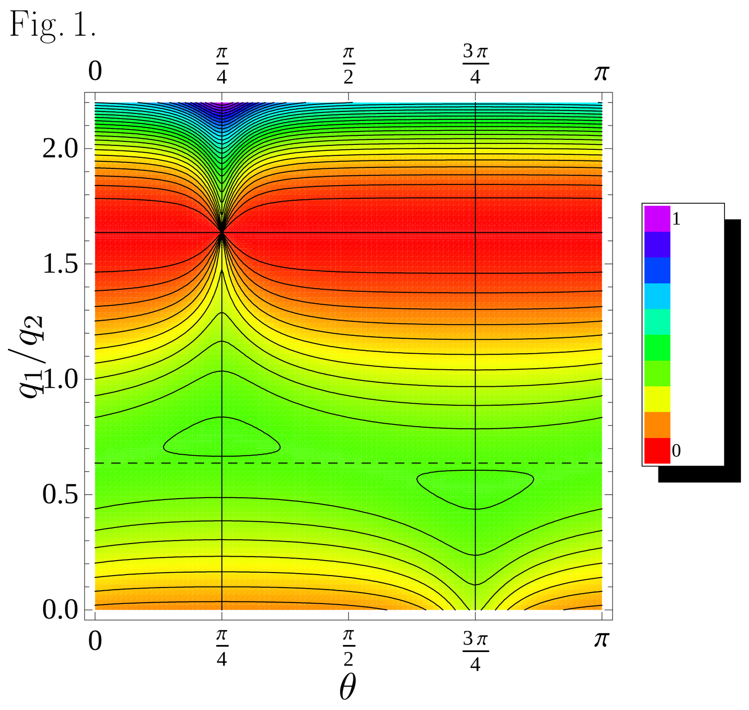


Figure 1: Dependence of the W^2 coefficient in Eq. (8) on the lateral rotation (θ). The absolute minimum is found for $\alpha_{x1}(\theta=0)=-q_2$ (here: $q_1/q_2=1.63$) and for different SO strength we find the minimum at $\theta=(1/4+n)\pi,\ n\in\mathbb{Z}$ if $q_1<(qs3/\sqrt{2})$ (dashed line: $q_1=(qs3/\sqrt{2})$) and at $\theta=(3/4+n)\pi,\ n\in\mathbb{Z}$ else. Here we set $q_{s3}=0.9$. The scaling is arbitrary.

- **Spin Dephasing**

The eigenvector of $H_c(\theta)$ which has the eigenvalue

$$E_1(k_x=0) = q_{s3}^2+q_{sm}^2 - \frac{(\alpha_{x1}^2+\alpha_{x2}^2-q_2^2)^2+\frac{qs3^2}{2}q_s^2}{12}W^2, \qquad (10)$$

is the triplet state $|S=1;m=0\rangle = (|\uparrow\downarrow\rangle+|\downarrow\uparrow\rangle)/\sqrt{2} \equiv |\rightrightarrows\rangle\hat{=}(0,1,0)^T$. This is equal to the z-component of the spin density whose evolution is described by the spin diffusion equation, Eq. (6). This gives an analytical description (Fig. 2) of numerical calculation done by J.Liu *et al.*, Ref. 4.

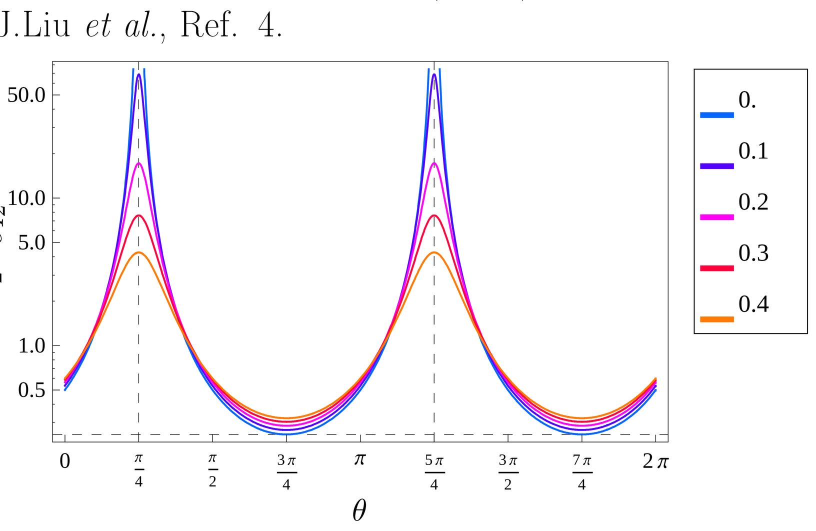


Figure 2: The spin dephasing time T_2 of a spin initially oriented along the [001] direction in units of $(D_e q_2^2)$ for the special case of equal Rashba and lin. Dresselhaus SOC. The different curves show different strength of cubic Dresselhaus in units of q_{s3}/q_2. In the case of finite cubic Dresselhaus SOC we set $W=0.4/q_2$. If $q_{s3}=0$: T_2 diverges at $\theta=(1/4+n)\pi,\ n\in\mathbb{Z}$ (dashed vertical lines). The horizontal dashed line indicated the 2D spin dephasing time, $T_2=1/(4q_2^2D_e)$.

Spin Relaxation Anisotropy in the (110) System

The Dresselhaus Hamiltonian with the confinement in $z\equiv$[110] direction ($\langle k_z\rangle=\langle k_z^3\rangle=0$, and $\langle k_z^2\rangle=\int|\nabla\phi|^2dz$) has the following form

$$H_{[110]} = -\gamma_D\sigma_z k_x\left(\frac{1}{2}\langle k_z^2\rangle - \frac{1}{2}(k_x^2-2k_y^2)\right). \qquad (11)$$

Including the Rashba SOC $q_2=2m_e\alpha_2$, noting that its Hamiltonian does not depend on the orientation of the wire, we end up with the following Cooperon Hamiltonian

$$\frac{C^{-1}}{D_e} = (Q_x-\tilde{q}_1S_z-q_2S_y)^2+(Q_y+q_2S_x)^2+\frac{\tilde{q}_3^2}{2}S_z^2, \qquad (12)$$

$$\text{with } \tilde{q}_1 = 2m_e\frac{\gamma_D}{2}\langle k_z^2\rangle - \frac{\gamma_D}{2}\frac{m_eE_F}{2}, \qquad (13)$$

$$\tilde{q}_3 = (3m_eE_F^2(\gamma_D/2)). \qquad (14)$$

⇒ In 2D states polarized in the z-direction have vanishing spin relaxation as long as we have no Rashba SOC.

- **Spin Relaxation in the Wire**

Again we apply appropriate Neumann *boundary condition*

$$(-\mathrm{i}\partial_y+2m_e\alpha_2S_x)C\left(x,y=\pm\frac{W}{2}\right)=0,\ \ \forall x \qquad (15)$$

and solve the Cooperon equation:

- Special case: without cubic Dresselhaus SOC

The lowest spin relaxation rate is found at *finite wave vectors* $k_{x_{\min}}=\pm\frac{\Delta}{24}(24-(q_2W)^2)$,

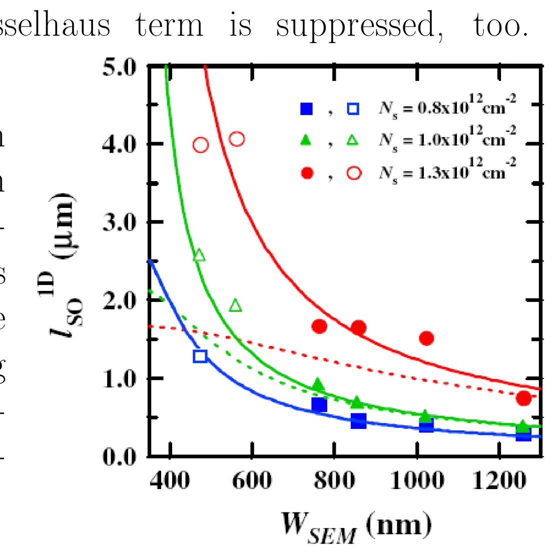
$$\frac{1}{D_e\tau_s} = \frac{(\tilde{q}_1^2+q_2^2)}{24}(q_2W)^2. \qquad (16)$$

As in the 2D case the spin relaxation rate vanishes for vanishing Rashba SOC $\Longrightarrow$ **there is no width dependence of weak localization/weak antilocalization in the case without Rashba SOC**.
If cubic Dresselhaus SOC cannot be neglected, the absolute minimum of spin relaxation can also shift to $k_{x_{\min}}=0$ (see solution in Ref. 3).

Diffusiv-Ballistic Crossover

For every direction in the diffusive (001) system there is still a finite spin relaxation, Eq. (7) and (8), due to cubic Dresselhaus SOC, at wire widths $W \ll L_{\text{SO}}$ (spin precession length L_{SO}). Experiments, e.g. the work by Kunihashi *et al.*, Ref. 5, however, show that in wires which do not fulfill the condition $l_e < W$, the cubic Dresselhaus term is suppressed, too.

In the right Fig. we show width dependence of the spin relaxation length l_{SO}^{1D} of different carrier density[5]. Solid lines and dashed lines show the l_{SO}^{1D} calculated from the theory by S.K.[1], with neglecting cubic Dresselhaus term and taking into account full SOIs, respectively.



To explain this one has to assume a finite number of transverse channels $N=k_FW/\pi$ in the q space, Eq. (2). The Cooperon to diagonalize reads then $(m_e\equiv 1)$

$$\frac{C^{-1}}{D_e} = 2f_1\left(Q_y+2\alpha_2S_x+2\left(a_1-\gamma_Dv^2\frac{f_3}{f_1}\right)S_y\right)^2$$
$$+2f_2\left(Q_x-2\alpha_2S_y-2\left(a_1-\gamma_Dv^2\frac{f_3}{f_2}\right)S_x\right)^2$$
$$+8\gamma_D^2v^4\left[\left(f_4-\frac{f_3^2}{f_2}\right)S_x^2+\left(f_5-\frac{f_3^2}{f_1}\right)S_y^2\right], \qquad (17)$$

with functions f_i which depend on N. The suppression of the cubic Dresselhaus term, also depending on Rashba SOC, is plotted in Fig. (3).

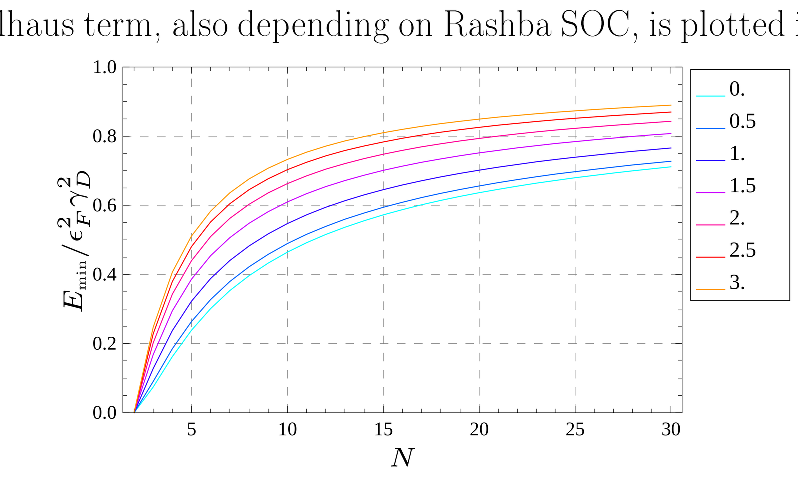


Figure 3: The lowest eigenvalues of the confined Cooperon Hamiltonian Eq. (17), equivalent to the lowest spin relaxation rate, are shown for $Q=0$ for different number of modes $N=k_FW/\pi$. Different curves correspond to different values of α_2/q_s, $q_s=\sqrt{(2\alpha_2)^2+(\gamma v_F^2/2-2\alpha_1)^2}$.

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