Dimensional Control of Spin-Relaxation and Antilocalisation in Quantum Wires

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Abstract

- As an introduction, we investigate first the effect of spin-orbit coupling (SOC) in ballistic quantum wires with Dirichlet boundary conditions and solve this boundary problem analytically. A non-abelian gauge transformation simplifies considerably that problem.
- In order to study **antilocalisation and the spin relaxation length in diffusive quantum wires with Rashba SOC**, we solve the Cooperon equation with spin and charge conserving boundary conditions (Neumann) [1]. Also here, a non-abelian gauge transformation turns out to be essential for an exact diagonalization of the Cooperon in the confined wire. This allows a comparison with previous results where only the transverse zero mode of the Cooperon equation has been taken into account [2]. As a result we confirm that **the spin relaxation rate becomes suppressed when the wire width is smaller than the bulk spin precession length** [2], resulting in a change from weak anti- to weak localisation. Surprisingly, **the suppression of spin relaxation rate is non- monotonous** but becomes first enhanced for wire widths on the order of the bulk spin precession length before it becomes diminished for smaller wire widths. It is smallest at the edge of the wire. The identical spin relaxation spectrum is obtained from a solution of the **quasiclassical spin diffusion equations**.

Getting started: Ballistic wire with Dirichlet boundary conditions

Hamiltonian with Rashba SOC

$$H = -\frac{1}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\mathbb{1} + \mathrm{i}\alpha_2\left(\boldsymbol{\sigma}_y\frac{\partial}{\partial x}-\boldsymbol{\sigma}_x\frac{\partial}{\partial y}\right) \quad (1)$$

with the boundary condition $\psi(x)|_{\pm\frac{W}{2}} \stackrel{!}{=} 0$.
We solved the problem analytically with the following Ansatz:

$$\psi = \mathrm{e}^{\mathrm{i}k_y y}\left[a\ \mathrm{e}^{\mathrm{i}\gamma}\mathrm{e}^{\mathrm{i}\xi x}\begin{pmatrix}\mathrm{e}^{\mathrm{i}\varphi_\xi}\\ 1\end{pmatrix} + h.c. + b\ \mathrm{e}^{\mathrm{i}\beta}\mathrm{e}^{\mathrm{i}\zeta x}\begin{pmatrix}-\mathrm{e}^{\mathrm{i}\varphi_\zeta}\\ 1\end{pmatrix} + h.c.\right] \quad (2)$$

Numerical solution: The non-abelian transformation
$U = \exp\left(\mathrm{i}\boldsymbol{\sigma}_y c x\right)$ simplifies the exact diagonalization, Fig.(1).

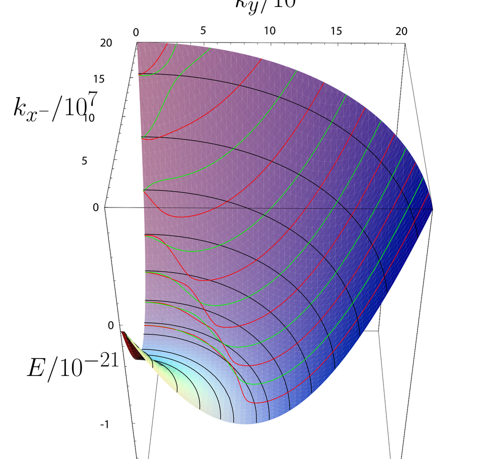

Figure 1: Projection of the spectrum for the wire with Dirichlet boundary conditions onto the outer free-spectrum paraboloid, $\frac{W}{L_{SO}}/\pi = 10.5$.

Cooperon Hamiltonian

The weak localisation correction to the conductivity is given by

$$\Delta\sigma = -\frac{2e^2}{2\pi}\frac{D}{\mathrm{Vol.}}\sum_{\mathbf{Q}}\sum_{\alpha,\beta=\pm}\mathcal{C}_{\alpha\beta\beta\alpha,\omega=0}(\mathbf{Q}), \quad (3)$$

where $\alpha,\beta=\pm$ are the spin indices, and the Cooperon propagator $\hat{\mathcal{C}}$ is for $\epsilon_F\tau \gg 1$ (ϵ_F, Fermi energy), given by

$$\hat{\mathcal{C}}_{\omega=E-E'}(\mathbf{Q}=\mathbf{p}+\mathbf{p}') = \tau\left(1-\sum_{\mathbf{q}} [E,\mathbf{p}+\mathbf{q};\ E',\mathbf{p}'-\mathbf{q}]\right)^{-1}, \quad (4)$$

Expanding $\hat{\mathcal{C}}$ to lowest order in the generalized momentum $\hat{\tilde{\mathbf{Q}}}$ leads to

$$\hat{\mathcal{C}}(\mathbf{Q}) = \frac{1}{D(\mathbf{Q}+2e\mathbf{A}+2e\mathbf{A}_{\mathbf{S}})^2+H_\gamma} \quad (5)$$

e.g. in GaAs (001), with the Rashba parameter α_2 and the Dresselhaus bulk coefficient γ

$$\mathbf{A}_{\mathbf{S}} = \frac{m_e}{e}\hat{a} = \begin{pmatrix}-\gamma\langle k_z^2\rangle+\gamma k_y^2 & -\alpha_2\\ \alpha_2 & \gamma\langle k_z^2\rangle-\gamma k_x^2\end{pmatrix}.\mathbf{S} \quad (6)$$

$$H_\gamma = (m_e^2\gamma\epsilon_F)^2(S_x^2+S_y^2) \quad (7)$$

with $S=\frac{1}{2}(\sigma+\sigma')$. As an example we chose pure Rashba with $Q_{so} = 1/L_{SO} = 2m_e\alpha_2$. The Hamiltonian $H_{\mathcal{C}} \equiv \frac{\hat{\mathcal{C}}^{-1}}{D}$ **decouples into a singlet and triplet sector** in the $\{|S=0;m=0\rangle,|S=1;m=\pm1\rangle\}$-representation. The eigenvalues of of in the 2D system are found to be (Fig.(2))

$$E_S = \mathbf{Q}^2 \quad (8)$$
$$E_{T_0} = \mathbf{Q}^2+Q_{so}^2 \quad (9)$$
$$E_{T_\pm} = \mathbf{Q}^2+\frac{3}{2}Q_{so}^2 \pm \frac{Q_{so}^2}{2}\sqrt{1+\left(\frac{4\mathbf{Q}}{Q_{so}}\right)^2} \quad (10)$$

where the energy of the singlet-state is denoted as E_S and the triplet states as E_T. Note that in the free system the minima are shifted to $k_x = \pm\frac{\sqrt{15}}{4}Q_{so}$ with $E=\frac{7}{16}Q_{so}^2$.

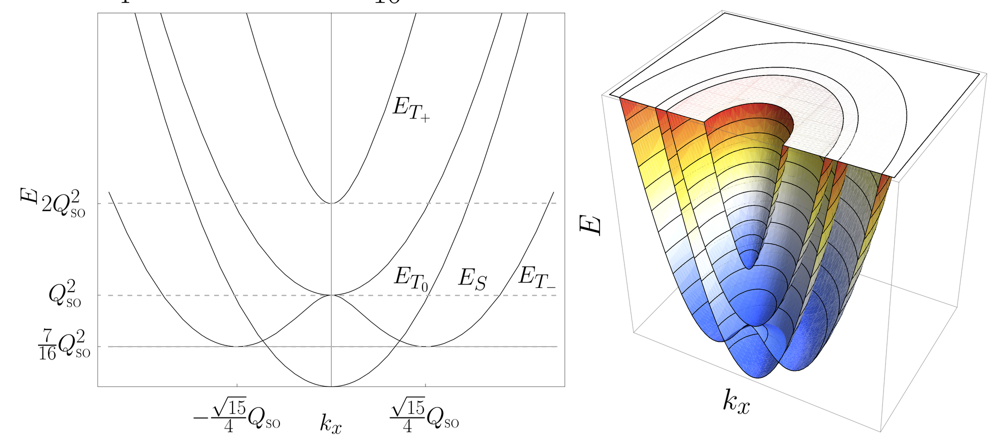

Figure 2: 2D spectrum of $H_{\mathcal{C}}$.

Modified Neumann Boundary Problem Boundary Problem

We apply spin and charge conserving BC:

$$\mathbf{j}_y(\omega,\mathbf{r})|_{y=\pm\frac{W}{2}} = 0$$
$$\sum_{\gamma\gamma'\delta\delta'}\hat{e}_y(\mathbf{Q}+\hat{a}2\mathbf{S})_{\gamma\gamma'}\mathcal{C}_{\gamma'\delta\delta'\gamma}(\omega,\mathbf{Q},\sigma,\sigma')|_{y=\pm\frac{W}{2}} = 0 \quad (11)$$

We set the strong condition that every term in the sum of the lhs of the last Eq. should vanish. To solve it: Simplify this BC while doing a **non-Abelian gauge transformation U transverse to the boundary**!

$$U^\dagger\left(-\mathrm{i}\frac{\partial}{\partial y}+2e(A_S)_y\right)UU^\dagger\mathcal{C}U\,|_{\pm\frac{W}{2}} \stackrel{!}{=} -\mathrm{i}\frac{\partial}{\partial y}\tilde{\mathcal{C}}\,|_{\pm\frac{W}{2}} = 0 \quad (12)$$

which is fulfilled by $\boxed{U=\exp(-\mathrm{i}Q_{so}S_x y)}$, so that one obtains the usual Neumann BC. The transformed Hamiltonian $\tilde{H}_{\mathcal{C}}$ can be written as

$$\tilde{H}_{\mathcal{C}} = \mathbf{Q}^2 - 2Q_{so}Q_x(\cos(Q_{so}y)S_y - \sin(Q_{so}y)S_z) + Q_{so}^2(\cos^2(Q_{so}y)S_y^2+\sin^2(Q_{so}y)S_z^2 - \sin(Q_{so}y)\cos(Q_{so}y)(S_yS_z+S_zS_y)) \quad (13)$$

Exact Diagonalization

To solve the Neumann boundary problem we use as Ansatz standing wave-functions transversal to the wire and free wave functions along the wire as applied to the Dirichlet boundary problem.
Taking into account only transverse zero modes, the resulting quasi-1D Hamiltonian is diagonalised exactly, yielding one singlet and three triplet Eigenvalues

$$E_S = K_x^2 \quad (14)$$
$$E_{T_0} = \frac{1}{2}+K_x^2-\frac{\sin(Q_{so}W)}{2Q_{so}W} \quad (15)$$
$$E_{T_\pm} = \frac{3}{4}+K_x^2+\frac{\sin(Q_{so}W)}{4Q_{so}W} - \frac{\sqrt{128(1-\cos(Q_{so}W))K_x^2+(Q_{so}W-\sin(Q_{so}W))^2}}{4Q_{so}W} \quad (16)$$

with $K_x \equiv Q_x/Q_{so}$. Going beyond zero-mode diagonalization and including many transverse modes

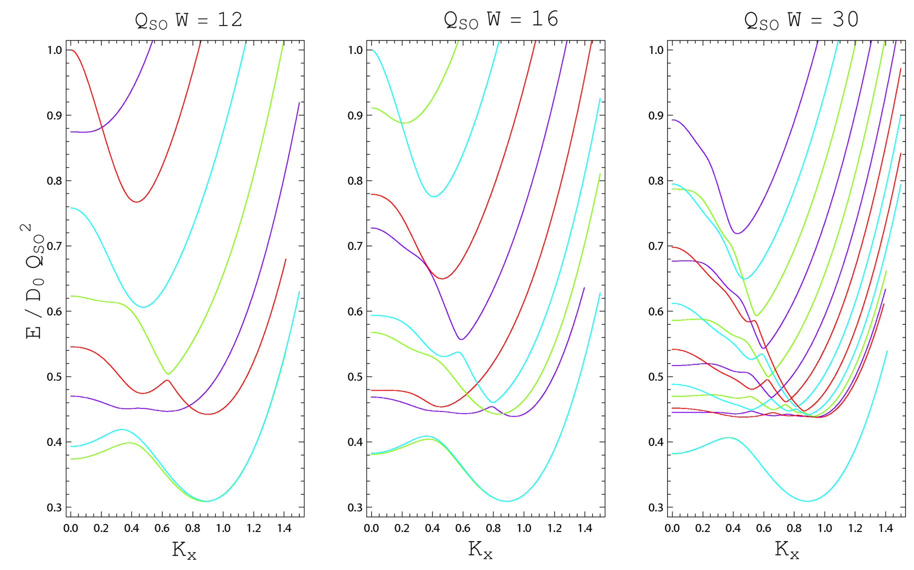

Figure 3: Spectrum of the Cooperon for different $Q_{so}W$ values, without singlet-modes E_S. For $Q_{so}W \gg 3$ and $E < \frac{7}{16}Q_{so}^2$ the **two boundary modes** can be seen. Their minimum form the absolute minimum of the spectrum. Due to **time-reversal-symmetry**: Identical to quasiclassical **spin-diffusion-dispersion.**

we find that
- the absolute energy minima are dominated by **boundary-modes**, located at $E<\frac{7}{16}Q_{so}^2$ (Fig.(3))
- the absolute energy minima$\sim\frac{1}{\tau_s}$ change non-monotonous with W (Fig.(5))
- and are located at $|k_x|>0$: compare Fig.(5) and Fig.(4)

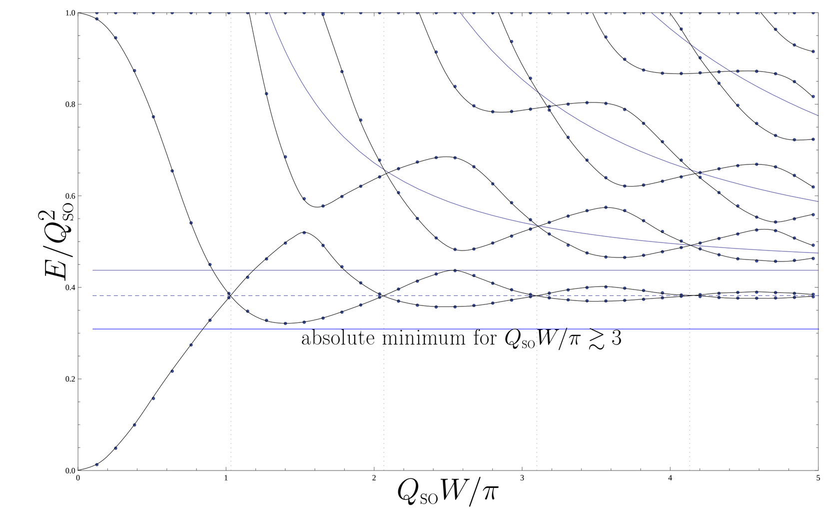

Figure 4: Energy at $K_x=0$ of the lowest modes plotted against $Q_{so}W/\pi$. Dotted vertical lines: $\left(\pi\frac{N}{\frac{\sqrt{15}}{4}}\right)$, $N\in\mathbb{N}$. Blue lines above $\frac{7}{16}Q_{SO}^2$: $\frac{7}{16}Q_{SO}^2+\left(\frac{N}{Q_{SO}W/\pi}\frac{\sqrt{15}}{4}\right)^2Q_{SO}^2$. Compare to [3].

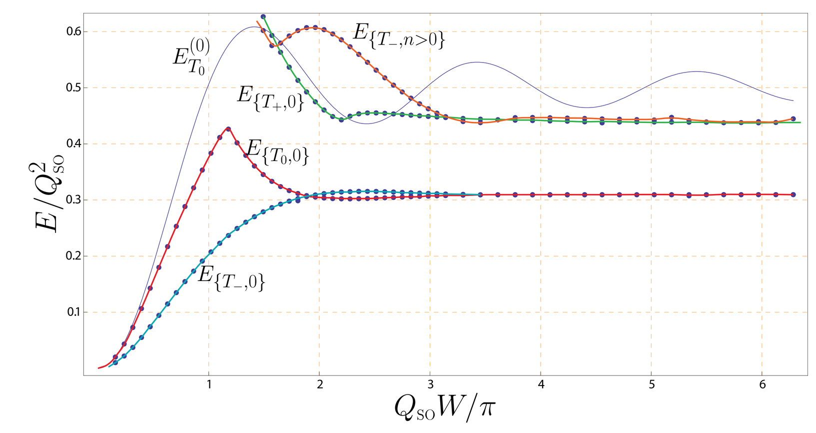

Figure 5: Absolute minima of the modes $E_{\{T_0,0\}}$, $E_{\{T_-,0\}}$, $E_{\{T_+,0\}}$ plotted against $Q_{so}W/\pi$. For comparison: The solution of the zeroth approximation for E_{T_0}, $E_{T_0}^{(0)}$ is shown.

For $Q_{so}W \ll 1$ we can integrate over Q_x analytically, and get [2]

$$\Delta\sigma = \frac{\sqrt{H_W}}{\sqrt{H_\varphi+B^*(W)/4}} - \frac{\sqrt{H_W}}{\sqrt{H_\varphi+B^*(W)/4+H_S(W)}} - 2\frac{\sqrt{H_W}}{\sqrt{H_\varphi+B^*(W)/4+H_S(W)}} \quad (17)$$

in units of $e^2/2\pi$, with $H_W=\frac{1}{4eW^2}$, the effective external magnetic field $B^*(W)=(1-1/(1+\frac{W^2}{3l_B^2}))B$ and the spin relaxation field $H_S(W)=\frac{1}{12}(Q_{so}W)^2m_e^2\alpha_2^2/e$.
For general combinations of linear Dresselhaus [001], α_1 and linear Rashba α_2 in the **D'yakonov-Perel'-spin-relaxation-regime**, one gets for $Q_{so}^2W^2=(q_R^2+q_D^2)W^2\ll 1$ with $q_{R/D}=2m_e\alpha_{2/1}$,

$$\frac{1}{\tau_S}(W) = \frac{D}{12}W^2\mid q_S^4-q_D^4\mid \quad (18).$$

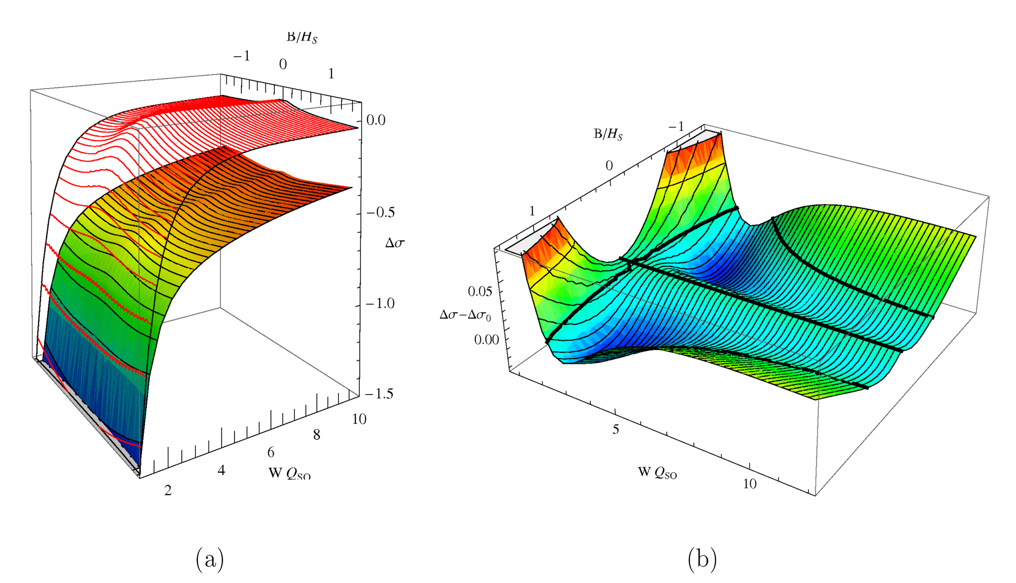

Figure 6: Magnetoconductivity **(a)** $\Delta\sigma(B)$ and **(b)** $\Delta\sigma(B)-\Delta\sigma(0)$ in units of e^2/π as function of magnetic field B (scaled with bulk relaxation field H_s), and the wire width W scaled with spin-orbit length L_{SO}, for pure Rashba coupling and cutoffs $1/Q_{SO}^2D_0\tau_\varphi = 0.08$, $1/\sqrt{Q_{SO}^2D_0\tau}=2$. Bold black lines indicate $\Delta\sigma(B)-\Delta\sigma(0)\equiv 0$.

Outlook

- including Dresselhaus-terms (direction dependence!) and external magnetic field
- WL/WAL in Graphene.

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