

Summer School on  
Rigidity theorems in  $\mathbb{A}^1$ -homotopy theory  
Regensburg, August 1-5, 2011

Organizers: U. Jannsen, N. Naumann

**Aims and Scopes:**

The summer school is intended to offer strong PhD students the opportunity to learn from leading researchers in the field about the recent advances due to F. Morel on the Friedlander Milnor-conjecture.

The prerequisites are moderate as there will be a number of introductory courses but prior exposition to some basic algebraic geometry and (simplicial) homotopy theory will be assumed.

**Application and funding:**

Application is by email to the following address:

brigitte.lindner@mathematik.uni-regensburg.de

Application is open now until all available places will be assigned. Please send a short CV and indicate what support you need. We will acknowledge receipt of your application within a few days and send out final confirmations in due course.

We expect to be able to cover both travel and housing expences for a substantial number of participants - thanks to the support of

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**Speakers:**

F. Morel (LMU Munich)

J. Ayoub (Zürich)

A. Asok (USC)

J. Fasel (Lausanne/Munich)  
L.-F. Moser (LMU Munich)  
G. Quick (Harvard/Münster)  
M. Spitzweck (Regensburg)  
M. Wendt (Freiburg)  
P.-A. Østvær (Oslo)

### Lecture Courses

The central events of this summer school will be the following lecture courses of 2-3 hours each. Additionally there will be ample time for informal discussions.

1.  $\mathbb{A}^1$ -geometry G. Quick

This course will provide an introduction to  $\mathbb{A}^1$ -homotopy theory as developed by Morel and Voevodsky [MV99],[Mor04].

The aim should be to have precise statements and motivating examples like the representability of algebraic  $K$ -theory while only discussing technicalities as far as time permits.

2. The stable  $\mathbb{A}^1$ -connectivity theorems P.-A. Østvær

This course should provide a reasonably complete proof of Morel's stable connectivity theorems from [Mor05].

3. Classical Rigidity J. Ayoub

This will be an account of the rigidity results of Suslin and Gabber for presheaves with transfer, following for example [MVW06], chapter 7. It's certainly instructive to see the well-known consequences  $K_*(k, \mathbb{Z}/n) \simeq K_*(L, \mathbb{Z}/n)$  and  $H^*(BGL(k), \mathbb{Z}/l) \simeq H_{\acute{e}t}^*(BGL, \mathbb{Z}/l)$  as for example in Jardine's notes, available at <http://www.math.uwo.ca/jardine/papers/ktheory/lecture010.pdf>. It will be important for later reference to have Theorem 4.3 of the original work of Suslin and Voevodsky [SV96] available.

4. The affine Brown-Gersten property for  $\text{Sing}^{\mathbb{A}^1}(G)$  L.-F. Moser

This course will explain the main ideas of the proof that the Suslin-Voevodsky

construction on an affine algebraic group possesses the Brown-Gersten property.

5. The result of Friedlander and Mislin A. Asok  
This course will explain the proof due to Friedlander and Mislin [FM84] of the full Friedlander-Milnor conjecture over (the algebraic closure of) a finite field.

6. The unstable connectivity theorems M. Spitzweck  
This course will give an exposition of the main ideas involved in the proof of Morel's unstable connectivity theorems from [Mora].

6 $\frac{1}{2}$ . Gersten-Rost-Schmid complexes J. Fasel  
This course will provide technical results needed for talk number 6.

7. Homotopy invariance of group cohomology M. Wendt  
Let  $k$  be a separably closed field,  $G/k$  a split semi-simple algebraic group of rank at least 2 and  $l$  a prime different from the characteristic of  $k$ . It is conjectured that for all  $n \geq 1$  the canonical map of graded algebras

$$H^*(G(k[t_1, \dots, t_n]), \mathbb{Z}/l) \rightarrow H^*(G(k), \mathbb{Z}/l)$$

is an isomorphism. This course will be an introduction to this problem leading up to the most recent results and will explain their relevance for the Friedlander-Milnor-conjecture.

8. Rigidity of  $\mathrm{BSing}_{\bullet}^{\mathbb{A}^1}(G)$  F. Morel  
This is the key lecture course of the entire summer school. Here, F. Morel will explain his recent work showing the rigidity of the Suslin-Voevodsky construction on a linear algebraic group [Morb].

Schedule:

All lectures take place in M??? on the first floor  
Discussion/tea and coffee is to be found in M201 on the second floor.

	Mo	Tue	We	Thu	Fr
9.00-10.00	Quick, I	Østvær, I	Morel, I	Morel, II	Morel, III
10.00 - 11.00			COFFEE		
11.00-12.00	Ayoub, I	Moser, II	Asok, III	Spitzweck, I	Spitzweck, II
12.00-13.30			LUNCH		
13.30-14.15	Quick, II	Asok, I	Wendt, I	Fasel, I	Fasel, II
14.45-15.30	Ayoub, II	Østvær, II	excursion	Wendt, II	Wendt, III
16.00-16.45	Moser, I	Asok, II		Question Session	Question Session
17.00			TEA		

## References

- [FM84] Eric M. Friedlander and Guido Mislin, *Cohomology of classifying spaces of complex Lie groups and related discrete groups*, Comment. Math. Helv. **59** (1984), no. 3, 347–361. MR 761803 (86j:55011)
- [Mora] Fabien Morel,  *$\mathbb{A}^1$ -algebraic topology over a field*, available at: <http://www.mathematik.uni-muenchen.de/~morel/A1homotopy.pdf>.
- [Morb] ———, *On the friedlander-milnor conjecture for groups of small rank*, available at: <http://www.mathematik.uni-muenchen.de/~morel/Harvardconf1.pdf>.
- [Mor04] ———, *An introduction to  $\mathbb{A}^1$ -homotopy theory*, Contemporary developments in algebraic K-theory, ICTP Lect. Notes, XV, Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2004, pp. 357–441 (electronic). MR 2175638 (2006m:19007)
- [Mor05] ———, *The stable  $\mathbb{A}^1$ -connectivity theorems*, K-Theory **35** (2005), no. 1-2, 1–68. MR 2240215 (2007d:14041)
- [MV99] Fabien Morel and Vladimir Voevodsky,  *$\mathbb{A}^1$ -homotopy theory of schemes*, Inst. Hautes Études Sci. Publ. Math. (1999), no. 90, 45–143 (2001). MR 1813224 (2002f:14029)

- [MVW06] Carlo Mazza, Vladimir Voevodsky, and Charles Weibel, *Lecture notes on motivic cohomology*, Clay Mathematics Monographs, vol. 2, American Mathematical Society, Providence, RI, 2006. MR 2242284 (2007e:14035)
- [SV96] Andrei Suslin and Vladimir Voevodsky, *Singular homology of abstract algebraic varieties*, *Invent. Math.* **123** (1996), no. 1, 61–94. MR 1376246 (97e:14030)