Seminar on cohomolgy operations and homotopy theory

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Time end place	:	Thursday, 4-6 pm, Seminarraum A, Wegelerstr. 10
Start	:	October, 16^{th}, 2008
Audience	:	Advanced topology students
Prerequisites	:	Basic algebraic topology, e.g. $K(\pi, n)$s, spectral sequences, the Steenrod algebra, Talk 13 uses some properties of spectra. If you are unfamiliar with some of these but willing to accept certain foundational results on faith, you can still participate. Don't hesitate to ask me for further reference, if needed.
Organisation	:	Below is a list of talks which will be distributed on a first come-first serve basis. If you want to give one of the talks, mail me (see above) indicating your name, e-mail adress and which talk you choose. The list of talks will be up-dated on the Seminar home-page (see above) indicating vacant/distributed talks.
Syllabus	:	The guiding motivation in this Seminar will be the computation of (stable) homotopy groups of spheres. We will read a large portion of the classical book of R. Mosher and M. Tangora [MT] on the subject. It starts with the main ideas of J-P. Serre's PhD which introduces the technique of approximating very concrete spaces, specifically the unit sphere S^n in euclidian n-space, by (geometrically) rather abstract spaces, called $K(\pi, n)$s, which are designed to serve homotopy theoretic needs. We follow up on the development of this theory by introducing the Adams spectral sequence and learning about some of its basic properties.

1) *Cohomology operations and the mod* 2 *Steenrod algebra* (16.10.08, N.N.) Review [MT], Chapter 1 omitting all proofs. Explain the basic properties of the Steenrod algebra ([MT], Chapter 3, Theorem 1) without proof. Illustrate the structure of the mod 2 Steenrod algebra by carrying through some of the computaions of [MT], p. 31 in detail.

key words: Cohomology operations, $K(\pi, n)$s, obstruction theory, Steenrod algebra.

2) *The Hopf Invariant* (23.10.08, N.N.)
Give a detailed account of all results in [MT], Chapter 4 omitting Proposition 3.
key words: Hopf invariant, indecomposables in the Steenrod algebra.

3) *Vector fields on spheres* (30.10.08, N.N.)
Review the geometry of (real) Grassmannians and Stiefel manifolds, i.e. [MT], Chapter 5 through corollary 1, with only sketches of proof. Explain the ensuing computaions of the mod 2 cohomology of these spaces, i.e. [MT], p.43 in detail and prove theorem 2.
key words: cell decomposition of Grassmannians, non-existence of vector fields on spheres.

4) *The Steenrod algebra* (6.11.08, N.N.)
Present in detail [MT], Chapter 6 through Theorem 3, paving the way for J. Milnor's results on the dual Steenrod algebra in the next talk.
key words: Hopf algebras and their duals, Steenrod algebra.

5) *The dual Steenrod algebra* (13.11.08, N.N.)
Discuss the results in [MT], Chapter 6 starting with Corollary 2 (p. 52). Include as many of the proofs as possible and solve exercise 1 (p. 58) and determine explicitely the dual of the Hopf algebra $\mathcal{A}$ introduced in this exercise.
key words: dual Steenrod algebra.

6) *Fibrations I* (20.11.08, N.N.)
The purpose of this talk is to see a basic computaional tool, the Serre spectral sequence, at work, [MT], Chapter 8. Make sure to thoroughly state all results but put emphasis on the examples, in particular the computation of $H_*(\Omega S^n)$ and $H^*(\Omega S^n)$ (p. 82). Give a complete proof of Proposition 1.
key words: Serre spectral sequence, transgression, Hurewicz Theorem.

7) *Cohomology of $K(\pi, n)s$* (27.11.08, N.N.)
Present [MT], Chapter 9 omitting the proofs of Proposition 1 and only

sketching the one of Theorem 2. Just state Proposition 3 through Theorem 4 on p. 90 but be sure to explain the proof of A. Borel's theorem in detail.
key words: Computaion of the mod 2 Steenrod algebra, A. Borel's theorem.

8) *Classes of abelian groups* (4.12.08, N.N.)
Present the results of [MT], Chapter 10 omitting the proof of Lemma 1. Make sure we see a detailed proof of Theorem 1 and be as detailed about the remaining proofs as time permits.
key words: Hurewicz and Whitehead modulo a class $\mathcal{C}$ of abelian groups, $\mathcal{C}_p$-approximation.

9) *Fibrations II* (11.12.08, N.N.)
The aim of this talk is to give a detailed proof of the Bockstein Lemma, [MT], Chapter 11, Theorem 1, in particular, verify the claim that $\iota_n \mapsto \iota_{n+1}$ on top of p. 104. Omit the proofs of Proposition 1 and 2 but try to include the proof of Lemma 1.
key words: Bockstein Lemma

10)+11) *Some homotopy groups of spheres* (18.12.08 and 8.1.09, N.N. and N.N.)
These two talks ripe the fruits of much of the previous work by computing $\pi_k^s(S^0)_{(2)}$ for $k \leq 7$. The two speakers must closely collaborate. Set the stage by explaining "stable stems" including the sketch proof of [MT], Chapter 12, Theorem 1. Then carefully explain the method of approximating S^n through a range of dimensions, looking at Figure 1 on p. 122 of [MT] may be helpful here. Now engage into the actual computation starting on p. 112 of [MT]. Do *not* try to present the computation of [MT], pp. 112-123 in complete detail but *do* include enough details to elucidate both the general method and its limitations. Time permitting, explain the unstable calculations of [MT], Chapter 12, Appendix.
key words: (stable) homotopy groups of spheres

12) *The Adams spectral sequence* (15.1.08, N.N.)
This talk consrtucts the Adams spectral sequence along the lines of [R], Chapter 2,1, all references below are to [R]. Include the elementary proof of 2.1.2, in particular part (g). Be short about the discussion on pp. 43-44 but introduce $E_\infty^{*,*}$. Prove 2.1.12 in detail. We should have a reasonably complete proof of 2.1.1 in the end and you should include example 2.1.19.
key words: Adams resolutions, Adams spectral sequence.

13) *The Adams spectral sequence II* (22.1.08, N.N.)
This talk illustrates how to work with the Adams spectral sequence. Compute $Ext^{0,*}$ and introduce the $h_i \in Ext^{1,2^i}$ as on p. 201 of [MT]. Then state theorem 3 on p. 203 of [MT] and explain the chart on p. 202 using the notation of [R], Figure A.3.1,a) (p.326). Your main task is to carry through the discussion starting on p. 203 of [MT] providing all the arguments in detail; we now have computed $\pi_k^s(S^0)_{(2)}$ for $k \leq 16$.
key words: the E_2-term and the first differential of the Adams spectral sequence.

14) *The May spectral sequence* (29.1.08, N.N.)
Explain the cobar complex of a Hopf algebra [R], A.1.2.11 and make it explicit through cohomological degree 3. Show that $\xi_1^{2^i} \in C^1_{A_*}$ is a cycle leading to the familiar elements $h_i \in Ext^{1,2^i}$. Introduce the filtration of a Hopf algebra by powers of its unit co-ideal [R], A.1.3.10 and the resulting (May-)filtration of $C^*_{A_*}$. Now explain without proof [R], Theorems 3.2.2 and 3.2.3 from P. May's PhD, then explain how to compute the May E_2-term displayed in [R], Figure 3.2.9 and perform this computation through dimension $t - s \leq 7$, say. Finally, proof in detail [R], 3.2.10 a),b) (for $j = 0$) and c). We have now verified at least part of the algebraic results used in the last talk.
key words: computation of the cohomology of a Hopf algebra

Referenzen

[MT] R. Mosher, M. Tangora, Cohomology operations and applications in homotopy theory, Harper and Row, 1968.

[R] D. Ravenel, Complex cobordism and stable homotopy groups of spheres, Pure and Applied Mathematics, 121. Academic Press, Inc., Orlando, FL, 1986.