Reading Course on J. Franke's "Uniqueness Theorems for certain triangulated categories with an Adams spectral sequence"

Dr. N. Naumann WS 08/09
Time and place: Wed, 10-12 am, Seminarraum C.

The aim of the seminar is to read the first two chapters of [F], leading up to a proof of the "abstract uniqueness theorem", [F], Theorem 5 (p. 56).
The final chapter 3 of [F] up-grades results of J. Morava to deduce that the E_n-local category at a prime p is algebraic, provided $2p-2 > n^2+n$. I will be happy to prepare talks on this material on demand.
Most of the talks below will not easily fit into 90 minutes but the speakers should just take their time.

1) (-, N.N.) *Systems of triangulated diagram categories* Recall the notions of (closed) model category, pointed model category and stable model category [Ho],[Hi] and introduce linear closed model categories, [F], 1.3.2 Def. 2. Compare these notions and give examples including [F], 1.3.3 (see section 3.1 for motivation). Now show the claim on top of p. 14, it seems advisable to check each axiom of a system of triangulated diagram categories (starting in 1.2) in turn for the example given by a linear model category. You may note that $\mathfrak{P}$ is properly contained in $\mathfrak{C}$, that $\mathfrak{C}$ is closed under exponentials and give instructive examples for "finite" and "finite-dimensional" categories.

2) (-, N.N.) *Properties of systems of triangulated diagram categories I* [F] 1.4 through 1.4.4: Give the construction of the natural transformations $(2)-(4)$ in 1.4.1 in detail. Work out the proof of Proposition 2. In Remark 1, note that $C^{(2)} \notin \mathfrak{P}$. In the proof of Proposition 3, b) you may take the existence of a suitable adjoint as the definition of (pre-)fibered. Give details for the proof of Proposition 4 and Corollary 2.

3) (-, N.N.) *Properties of systems of triangulated diagram categories II* [F] 1.4.5-1.5: For the proof of Theorem 1, recall the notion of a triangulated category. Check that $\mathfrak{P}$ and $\mathfrak{C}$ are stable under $\mathrm{Sub}(-)$. Discuss the convergence properties of the spectral sequence in Proposition 10. In the proof of Theorem 2, omit the part which refers to [Kel91] and work out the details of the proof of Theorem 3.

4) (-, N.N.) *Strong Linearity* [F] 1.6: Explain the details of the example on top of p. 45. Explain that diagram categories based on chain complexes always admit a strongly linear structure, c.f. Remark 1 on p. 66.

5) (-, N.N.) *Adams Spectral Sequence* [F], 2.1: Recall the construction of the spectral sequence (2) on p. 53, in particular, explain where the differentials go and discuss convergence. Try to prove the independence claim on p. 53, bottom, using the expected mapping property of resolutions as in (1), p. 53. Give a proof for the final clause in Proposition 1. Work out the proof of Proposition 3 and explain that $(\mathcal{D}^{([1],1)}(\mathcal{A}), H^0)$ (c.f. 1.3.3 on p. 14) is an example of a triangulated category with an Adams spectral sequence. You should include 3.2, Proposition 1 as a motivation.

6) (-, N.N.) *The uniqueness theorem* [F], 2.2: Explain why the case $N = 1$ is trivial on p. 57, bottom. Note that C_N is a finite category of dimension 1. (7) is to hold true for all $i \in \mathbb{Z}/\mathbb{N}\mathbb{Z}$, the shift needs to be interpreted suitably. Make the map α in (3) explicit. Check that (1.4.32) specializes to (14) as claimed. Work out the missing details leading to (15). Prove (16) and give details for the construction of the map (15) $\to$ (16). Explain why in (19) there appears rather $\mathcal{A}$ than $\mathcal{B}$. Give details for the proof of Proposition 2.

Referenzen

[F] Franke, Uniqueness Theorems for certain triangulated categories with an Adams spectral sequence, available at: http://www.math.uiuc.edu/K-theory/0139/

[Hi] Hirschhorn, Model categories and their localizations.

[Ho] Hovey, Model categories.