# Introduction to Stable Homotopy Theory
# 5. Übungsblatt

**Aufgabe 1**
Calculate the geometric realization of the simplicial set $\Delta[n]$ represented by the finite ordinal $[n] = \{0 < 1 < \cdots < n-1 < n\}$.

**Aufgabe 2**
Let $X$ be any nonempty topological space and consider the simplicial space $\Pi(X)$ obtained as the nerve of the covering $X \to *$ of the point by $X$ (i.e., in degree $n$, $\Pi(X)$ is the the product $X^{n+1}$ of $n+1$ copies of $X$, with the face and degeneracy maps given by partial diagonals and projections, respectively). Show that the geometric realization $|\Pi(X)|$ of $\Pi(X)$ is contractible.

**Aufgabe 3**
Let $(G, e)$ be a topological group with identity element $e$ such that the underlying pair of spaces $(G, e)$ is a relative CW complex (in other words, $e$ is a 0-cell of $G$). Let $X$ and $Y$ be CW complexes, let $q : Q \to Y$ be a principal $G$-bundle on $Y$, and consider the principal $G$-bundles $p_0 : f_0^*Q \to X$ and $p_1 : f_1^*Q \to X$ on $X$ associated to a pair of homotopic maps $f_0, f_1 : X \to Y$. Show that $f_0^*Q$ and $f_1^*Q$ are isomorphic as principal $G$-bundles on $X$. [Hint: a homotopy $h : I \times X \to Y$ defines a bundle $h^*Q$ over $I \times X$; show that $h^*Q$ must trivialize on a cover of $I \times X$ of the form $\{I \times U_\alpha\}$ for $\{U_\alpha\}$ a numerable open cover of $X$, and use the resulting trivializations to produce an isomorphic of bundles $f_0^*Q \cong f_1^*Q$.]

**Aufgabe 4**
Let $(G, e)$ be a topological group with identity element $e$ such that the underlying pair of spaces $(G, e)$ is a relative CW complex. Suppose that $X$ is a finite CW complex (that is, $X$ has is a CW complex with only a finite number of cells). Show that the function

$$[X, BG] \longrightarrow \mathrm{Bun}_G(X),$$

which sends a homotopy class of map $f : X \to BG$ to the isomorphism class of the principal $G$-bundle $f^*EG \to X$, is an isomorphism. [Hint: one can define a map back by noting that, for any principal $G$-bundle $P \to X$, there is a unique-up-to-homotopy $G$-equivariant map $P \to EG$.]

*Due: **22.6.2011** in the exercise session.*