

# Memory effects and thermodynamics in strong field plasmas

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We study the evolution of a strong field plasma using a quantum Vlasov equation with a non-Markovian source term and a simple collision term, and calculate the time dependence of the energy and number density, and the temperature. The evolution of a plasma produced with BNL RHIC-like initial conditions is well described by a low density approximation to the source term. However, non-Markovian aspects should be retained to obtain an accurate description of the early stages of a CERN LHC-like plasma.

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At extreme temperature, hadronic matter undergoes a phase transition to an equilibrated quark gluon plasma [1]. Estimates from phenomenological models and lattice-QCD indicate a critical temperature for this transition:  $T_c \sim 170$  MeV, which corresponds to an energy density of  $2\text{--}3$  GeV/fm<sup>3</sup> [2]. It is hoped that this plasma will be produced at the BNL Relativistic Heavy-Ion Collider (RHIC) and/or the CERN Large Hadron Collider (LHC) [3]. However, little is currently known about the formation, equilibration and hadronization of the plasma, and a variety of complementary tools and models are being employed to improve understanding of these aspects; e.g., Refs. [4–8]. Herein we employ a Vlasov equation, which preserves quantum coherence effects in the particle production process, and focus on the importance of memory (non-Markovian) effects in the pre-equilibrium phase of strong field plasmas.

We employ a flux-tube model to explore the creation of a strong-field plasma and follow its evolution towards equilibrium. The model assumes that the collision of two heavy nuclei produces a strong background field and a region of high energy density, which decays via pair emission. The particles produced in this process are accelerated by the background field, providing a current and a field that opposes the background field. This is the back-reaction process, which may result in plasma oscillations. For QED in an external field it has been studied via mean field methods and using a quantum Vlasov equation with a Schwinger-like source term [9]. Collisions between the particles damp the plasma oscillations and are necessary to equilibrate the plasma, and their effect has been modeled in the Vlasov equation approach [10–13].

For strong fields, non-Markovian aspects of the particle production mechanism are very important [14–18]. Herein we emphasize this, using a relativistic transport equation with the non-Markovian source term derived in Refs. [16,17,19] and explored in Refs. [18,20]. We apply it with impact energy densities of the scales anticipated at RHIC and LHC, and study the time evolution of the plasma's properties.

We model the effect of the nucleus-nucleus collision by an external, spatially-homogeneous, time-dependent Coulomb-gauge vector potential, which defines the longitudinal direction:  $A_\mu = [0, 0, 0, A(t)]$ . The kinetic equation describing fermion production in this external field is [19]

$$\frac{df(\vec{p}, t)}{dt} = S(\vec{p}, t) + C(\vec{p}, t) \quad (1)$$

where, in contrast to the Schwinger production rate, the source term here is momentum and time dependent:

$$S(\vec{p}, t) = \frac{1}{2} \frac{eE(t)\varepsilon_\perp}{\omega^2(t)} \int_{-\infty}^t dt' \frac{eE(t')\varepsilon_\perp}{\omega^2(t')} \times [1 - 2f(\vec{p}, t')] \cos[2(\Theta(t) - \Theta(t'))], \quad (2)$$

$E(t) = -dA(t)/dt$  and  $e$  is an electric charge, with the dynamical phase and total energy, respectively,

$$\Theta(t) = \int_{-\infty}^t dt' \omega(t'), \quad (3)$$

$$\omega(t) = \sqrt{\varepsilon_\perp^2 + (p_\parallel - eA(t))^2}, \quad (4)$$

where  $\varepsilon_\perp = \sqrt{m^2 + \vec{p}_\perp^2}$  is the transverse energy.  $m^2 \sim \Lambda_{QCD}^2 \sim 0.2$  GeV/fm sets a typical scale.

The second term on the right-hand-side (RHS) of Eq. (1) describes collisions between the particles, and we employ a simple and widely-used model

$$C(\vec{p}, t) = \frac{f^{eq}(\vec{p}, t) - f(\vec{p}, t)}{\tau_r}, \quad (5)$$

where  $\tau_r$  is the “relaxation time” and  $f^{eq}$  is the thermal equilibrium distribution functions for fermions:

$$f^{eq}(\vec{p}, t) = \frac{1}{\exp[\omega(\vec{p}, t)/T(t)] + 1}. \quad (6)$$

The temperature profile in Eq. (6) is *a priori* unknown and is determined by requiring that the average energy in our ensemble is that of a quasi-equilibrium fermion gas, Eqs. (15)–(17). We note that the relaxation time approximation is only valid under conditions of local “quasi-equilibrium.” That is difficult to justify in the presence of strong fields. Further, a realistic collision term is likely to generate memory effects

additional to those present in the source term, which are our present focus. The improvement of the collision term is therefore an important contemporary challenge.

The kinetic equation, Eq. (1), is non-Markovian for two reasons: (i) the source term on the RHS requires knowledge of the entire history of the evolution of the distribution function from  $t_{-\infty} \rightarrow t$ ; and (ii) even in the low density limit [ $f(t) \approx 0$ ], the integrand is a non-local function of time as is apparent in the coherent phase oscillation term:  $\cos[\Theta(t) - \Theta(t')]$ . The ideal Markov limit was derived in Ref. [16] and is reproduced for small fields in our approach. For our purpose, which is an elucidation of the importance of memory effects, it is enough to compare the full calculation with the low-density limit alone; i.e., we retain the cos-term but employ a source term that is independent of the distribution function

$$S^0(\vec{p}, t) = \frac{1}{2} \frac{eE(t)\epsilon_{\perp}}{\omega^2(t)} \times \int_{-\infty}^t dt' \frac{eE(t')\epsilon_{\perp}}{\omega^2(t')} \cos[2(\Theta(t) - \Theta(t'))]. \quad (7)$$

The kinetic equation, Eq. (1), with the source term of Eq. (7) has the general solution

$$f^0(\vec{p}, t) = \int_{-\infty}^t dt' \exp\left[\frac{t' - t}{\tau_r}\right] \left( S^0(\vec{p}, t') + \frac{f^{eq}(\vec{p}, t')}{\tau_r} \right). \quad (8)$$

The effect of back reactions on the induced field is accounted for by solving Maxwell's equation:  $\dot{E}(t) = -j(t)$ . The total electric field,  $E(t)$ , is the sum of an external field,  $E_{ex}(t)$ , and an internal field,  $E_{in}(t)$ . Herein we assume that the plasma is initially produced by the external field, excited by an external current,  $j_{ex}(t)$ , such as might represent a heavy ion collision

$$E_{ex}(t) = -A_0 [b \cosh^2(t/b)]^{-1}. \quad (9)$$

This model electric field “switches-on” at  $t \sim -2b$  and off at  $t \sim 2b$ , with a maximum magnitude of  $A_0/b$  at  $t=0$ . Once this field has vanished only the induced internal field remains to create particles and affect their motion.

Continued spontaneous production of charged particle pairs creates a polarization current,  $j_{pol}(t)$ , that depends on the particle production rate,  $S(\vec{p}, t)$ . Meanwhile the motion of the existing particles in the plasma generates a conduction current,  $j_{cond}(t)$ , that depends on their momentum distribution,  $f(\vec{p}, t)$ . The internal current is the sum of these two contributions

$$\dot{E}_{in}(t) = -j_{in} = -j_{cond}(t) - j_{pol}(t). \quad (10)$$

The renormalized Maxwell equation is [20]

$$\begin{aligned} \dot{E}_{in}(t) = & -2e \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{p_{\parallel} - eA(t)}{\omega(\vec{p}, t)} \left[ f(\vec{p}, t) \right. \\ & \left. + \frac{1}{2} \frac{\omega(\vec{p}, t)}{\dot{\omega}(\vec{p}, t)} \frac{df(\vec{p}, t)}{dt} - \dot{E}(t) \frac{\epsilon_{\perp}^2}{8\omega^4(p_{\parallel} - eA(t))} \right]. \end{aligned} \quad (11)$$

The first two terms on the RHS represent the conduction and polarization current, respectively, while the last term arises in regularizing the polarization current. The solution of the coupled pair, Eqs. (1) and (11), yields  $E(t)$  and  $f(\vec{p}, t)$ , and makes evident effects such as plasma oscillations and collisional damping [20].

All bulk thermodynamical properties are expressed in terms of  $f(\vec{p}, t)$ , and of particular interest herein are: the energy density

$$\epsilon_{\text{Total}}(t) = \frac{1}{2} E^2(t) + \epsilon(t), \quad (12)$$

$$\epsilon(t) = 2 \int \frac{d^3 \vec{p}}{(2\pi)^3} \omega(\vec{p}, t) f(\vec{p}, t), \quad (13)$$

with the pressure  $p(t) = 1/3 \epsilon(t)$ ; the particle number density

$$n(t) = 2 \int \frac{d^3 \vec{p}}{(2\pi)^3} f(\vec{p}, t); \quad (14)$$

and the temperature profile:  $T(t)$ , determined from the condition

$$\frac{\epsilon(t)}{n(t)} = \frac{\epsilon^{eq}(t)}{n^{eq}(t)}, \quad (15)$$

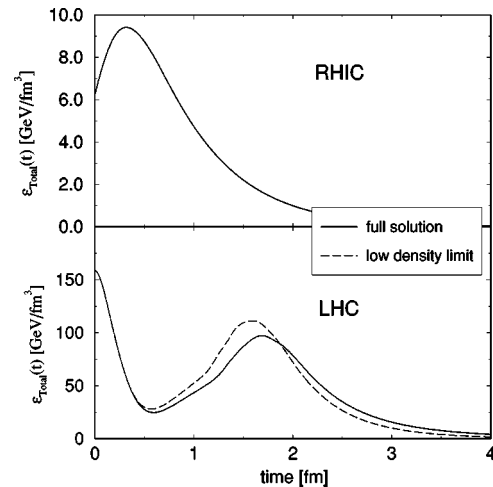


FIG. 1. Time evolution of the energy density for RHIC (upper panel) and LHC (lower panel) conditions.

$$\epsilon^{eq}(t) = \int \frac{d^3\vec{p}}{(2\pi)^3} \omega(\vec{p}, t) f^{eq}(\vec{p}, t), \quad (16)$$

$$n^{eq}(t) = \int \frac{d^3\vec{p}}{(2\pi)^3} f^{eq}(\vec{p}, t), \quad (17)$$

which implements our constraint that the average energy in the ensemble is equal to that of a quasi-equilibrium fermion gas.

To solve the coupled equations, we assume a trial form for  $E(t)$ ,  $T(t)$  and solve for  $f(\vec{p}, t)$  from Eq. (1).  $f(\vec{p}, t)$  so obtained yields an iterated  $E(t)$ ,  $T(t)$  from Eqs. (11), (15). The procedure is repeated until seed and iterate agree within 0.1%.

Our primary goal is to demonstrate that non-Markovian effects influence the early stage of plasma evolution. Therefore we contrast the results obtained in both the low-density limit, Eq. (7), and with the full source term, Eq. (2). The parameters characterizing the external field:  $b = 0.5/\Lambda_{\text{QCD}} \approx 0.5$  fm,  $A_0^{\text{RHIC}} = 4 \Lambda_{\text{QCD}}$  and  $A_0^{\text{LHC}} = 20 \Lambda_{\text{QCD}}$ , are chosen in order to obtain initial field energy densities of the magnitude expected in experiments, Fig. 1. We choose a relaxation time  $\tau_r = 1/\Lambda_{\text{QCD}} \approx 1$  fm.

In Fig. 1 we see that for RHIC conditions the energy density rises rapidly but, after reaching a maximum, decays monotonically. In this case the full solution and the solution obtained in the low density limit are quantitatively identical. For LHC conditions, with an initial energy density twenty-times larger, the situation is different. The solution obtained in the low density limit is only a qualitative guide to the plasma's behavior. The deviation between the two curves begins when the strength of the external field wanes. Furthermore under LHC-like conditions plasma oscillations are evident on observable time scales and lead to the appearance of a second maximum in the thermodynamic quantities. This effect retards the evolution to thermal equilibrium and the formation time is doubled compared to RHIC-like conditions. However, this effect is particularly sensitive to details of the collision term and observational conclusions must await improvements to the relaxation time approximation.

In Fig. 2 we present the time evolution of the electric field and particle number density for LHC conditions. The plasma oscillations are damped on a time scalar characteristic of the collision time. The evolution obtained in the low density limit again provides only a qualitative guide to the behavior of the complete solution. The plasma oscillation frequency and amplitude are larger in the low density limit because damping is less effective in the absence of the Pauli blocking factor:  $1 - 2f(\vec{p}, t)$ . This same effect is responsible for the overestimate of  $n(t)$  in the low density limit.

Our calculations also yield temperature profiles, which are depicted in Fig. 3. The RHIC-like source conditions yield an initial temperature  $T^{\text{RHIC}}(t=0) \sim 0.5$  GeV, and the temperature decreases monotonically with increasing  $t$ . The LHC-like source conditions yield an initial temperature twice as large:  $T^{\text{LHC}}(t=0) \sim 0.9$  GeV, and the temperature oscillates

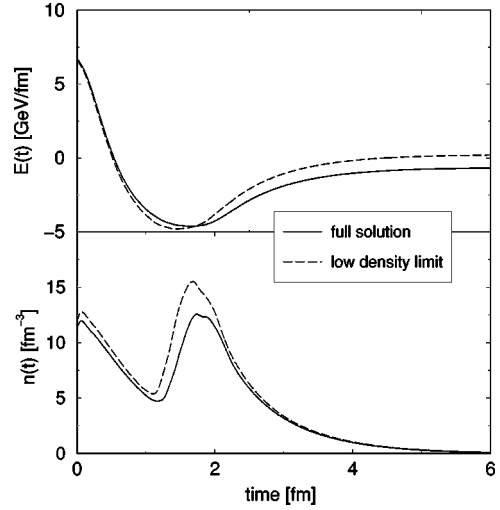


FIG. 2. Time evolution of the electric field (upper panel) and the particle density (low panel) for LHC conditions.

in tune with the energy density. In Ref. [20] the temperature profile was not determined self-consistently, instead an *ansatz* was used:

$$T(t) = T_{eq} + (T_m - T_{eq}) e^{-t^2/t_0^2}, \quad (18)$$

with  $T_{eq} = \Lambda_{\text{QCD}}$ ,  $T_m = 2 T_{eq}$ ,  $t_0^2 = 10/T_{eq}^2$ , which is also plotted in Fig. 3. It is evident that the *ansatz* provides a not unreasonable model of RHIC-like conditions.

We have solved a quantum Vlasov equation under conditions that qualitatively mimic those anticipated at RHIC and LHC. Under RHIC conditions the low density approximation to the source term provides an accurate description of the plasma's early stages. However, the non-Markovian features of the source term become important when the initial energy density is LHC-like and generate effects that are likely to be observable. Following this study we anticipate that the analogue of our non-Markovian source term would have a significant effect in a non-Abelian transport equation [13].

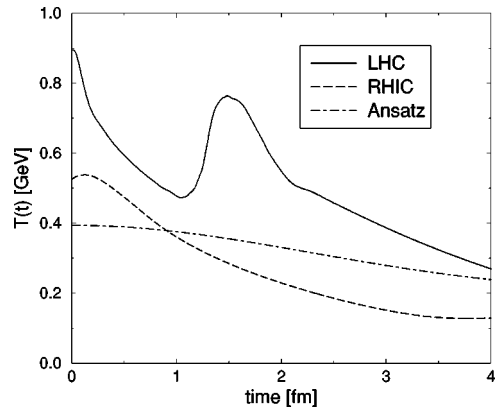


FIG. 3. Time evolution of the quasi-equilibrium temperature for RHIC and LHC initial conditions. For comparison we also plot the *ansatz* in Eq. (18).

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