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Strong QED with Weak Gauge Dependence: Critical Coupling and Anomalous Dimension

D. Atkinson¹, J.C.R. Bloch², V.P. Gusynin³, M.R. Pennington², M. Reenders¹

¹Institute for Theoretical Physics, University of Groningen, 9747 AG Groningen, The Netherlands

²Centre for Particle Theory, University of Durham, Durham DH1 3LE, U.K.

³Institute for Theoretical Physics, Ukrainian Academy of Sciences, 252143 Kiev, Ukraine



1 Introduction

Three years ago, one of us introduced an Ansatz for the full vertex function of quenched quantum electrodynamics (QED) [1]. This not only ensures satisfaction of the Ward-Takahashi identity and avoids singularities that would imply the existence of a scalar, massless particle, but it also respects the requirement of multiplicative renormalizability, a property of exact QED that is destroyed by the popular ladder or rainbow approximation. It agrees moreover with perturbative results in the weak coupling limit.

D. Atkinson

Institute for Theoretical Physics,
University of Groningen,
9747 AG Groningen, The Netherlands

J.C.R. Bloch

Centre for Particle Theory,
University of Durham,
Durham DH1 3LE, U.K.

V.P. Gusynin

Institute for Theoretical Physics,
Ukrainian Academy of Sciences,
252143 Kiev, Ukraine

M.R. Pennington

Centre for Particle Theory,
University of Durham,
Durham DH1 3LE, U.K.

M. Reenders

Institute for Theoretical Physics,
University of Groningen,
9747 AG Groningen, The Netherlands

In this paper we consider the Dyson-Schwinger equations in a general covariant gauge, with the Curtis-Pennington Ansatz, and apply bifurcation analysis to them. This involves calculating the Fréchet derivative of the nonlinear mapping of the mass function into itself. Thanks to the scale-invariance of the problem, the bifurcation equation can be solved by inspection, in the limit that the ultra-violet cut-off is taken to infinity. A solution for the mass-function is a power of the momentum that has to satisfy a certain transcendental equation, the power being directly related to the anomalous dimension of the $\bar{\psi}\psi$ operator. The onset of criticality is heralded by the coming together of two solutions of this transcendental equation, for that is the indication that oscillatory takes over from non-oscillatory behaviour. This study has been performed independently by the authors in two groups (AGR and BP). Obtaining common results, we have merged to present this work.

Abstract: We study chiral symmetry breaking in quenched QED₄, using a vertex Ansatz recently proposed by Curtis and Pennington. Bifurcation analysis is employed in a general covariant gauge to investigate the gauge-dependence of the critical coupling for chiral symmetry breakdown. This turns out to be relatively minor. The anomalous dimension of the composite operator $\bar{\psi}\psi$ is also calculated: it is slightly greater than one, in a neighbourhood of the Landau gauge, lending credence to the relevance of four-fermion operators.

2 Curtis-Pennington Equations

In Table 1 we have summarized the equations of the Curtis-Pennington Ansatz. The photon propagator is taken bare — the quenched approximation — with covariant gauge parameter ζ . The fermion propagator has the most general possible form, involving the mass-function, $\mathcal{M}(-p^2)$, and the wave-function, $\mathcal{Z}(-p^2)$ (these correspond respectively to Curtis and Pennington’s Σ and F).

The physical mass of the fermion is defined to be the lowest position at which the denominator function in the fermion propagator,

$$S_F(p) = \mathcal{Z}(-p^2) \frac{\gamma_\mu p^\mu + \mathcal{M}(-p^2)}{p^2 - \mathcal{M}^2(-p^2)},$$

has a zero, which is therefore a solution, m , of

$$m = \mathcal{M}(-m^2).$$

On physical grounds, this singularity should be on the real timelike axis of p^2 and should be gauge-independent; note that it would be a pole only if the photon were given a fictitious mass: with a massless photon, the singularity is a branch-point, the nature of which depends on the gauge. What Curtis and Pennington call ‘Euclidean mass’, namely the lowest solution of

$$\mathcal{M} = \mathcal{M}(M^2),$$

is not the same as the physical mass, m , and it is not expected to be exactly gauge-invariant. If one is going to abandon the attempt to calculate m , as one well might do in view of the Atkinson-Blatt complex branch-points [5], one might perhaps take $\mathcal{M}(0)$ as an *Ansatz effective mass*. At best one might hope it to be approximately gauge-invariant, on the grounds that it should be close to the physical mass m , which is gauge-invariant, at least in exact QED, or in a quenched approximation in which the first two Ward-Takahashi identities are respected [6].

The value of the wave-function at an arbitrarily selected renormalization point, μ , is defined to be the wave-function renormalization constant, which is conventionally dubbed Z_2 :

$$Z_2 = \mathcal{Z}(\mu^2).$$

It is convenient to choose the renormalization point to be Euclidean; the renormalized wave function is specified by

$$\tilde{\mathcal{Z}}(x) = Z_2^{-1} \mathcal{Z}(x). \quad (1)$$

The Curtis-Pennington Ansatz defines a renormalizable scheme, so that in it $\tilde{\mathcal{Z}}(x)$ has a finite limit as the ultra-violet regularization is removed. The renormalized wave-function contains no explicit cut-off, but it is dependent on the renormalization point, and on the gauge parameter.

Chiral symmetry breaking occurs if the coupling, α , is greater than a certain critical value, α_c . This critical coupling is potentially a physically measurable quantity, since it signals a change of phase, and so it should be gauge invariant. Although this is not exactly true in the Curtis-Pennington system, it is approximately so. Indeed, the requirement that α_c be gauge-invariant could perhaps be used to specify further the form of the Ansatz for the vertex function. In particular, the second term in the expression between the first set of parentheses {..} in the formula for Γ^a in Table 1 is not uniquely determined, and the above requirement might with profit be used to refine this transverse part of the vertex. The basic coupled integral equations are given in the third and fourth lines of Table 1, the complicated kernels I and J being explicit functions of \mathcal{M} and \mathcal{Z} . These equations were given in [2].¹

3 Bifurcation Equations

As can be seen from Table 1, the complete Curtis-Pennington equations are nonlinear and complicated. Clearly $\mathcal{M}(x) \equiv 0$ is always a possible solution; but it is not the one in which we are interested. However, the equations simplify at the critical point, where a nontrivial solution *bifurcates* away from the trivial one. To investigate this critical point, we have to take the Fréchet derivative of the nonlinear operators with respect to $\mathcal{M}(x)$ and evaluate it at the trivial ‘point’, $\mathcal{M}(x) \equiv 0$. This amounts in fact simply to throwing away all terms that are quadratic or higher in the mass function. It must be emphasized that this is *not* an approximation: it is a precise manner to locate the critical point by applying bifurcation theory.

¹They first appeared in [7] — note the misprints corrected in the erratum. In [7], the $\Lambda \rightarrow \infty$ limit was taken in a way that failed to respect axial current conservation, and so it was incorrectly deduced that chiral symmetry breaking occurs for all values of the coupling. This was rectified in [2].

Up to terms linear in $\mathcal{M}(x)$ and $\mathcal{M}(y)$, the kernels I and J reduce to

$$I(y, x) = -\xi \frac{y^2}{x^2} \theta(x - y) + \mathcal{O}(\mathcal{M}^2) \quad (2)$$

$$\begin{aligned} J(y, x) &= \frac{3}{2} \mathcal{M}(y) \left\{ 1 + \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} + \frac{y+x}{y-x} \left[1 - \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \right] \right\}_{x>} \\ &\quad - \frac{3}{2} x \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \frac{y^2}{x_2^2} + \xi \frac{\mathcal{Z}(y)}{\mathcal{Z}(x)} \mathcal{M}(y) \frac{y}{x} \theta(x-y) + \mathcal{O}(\mathcal{M}^3), \end{aligned} \quad (3)$$

where $x_> = \max(x, y)$, and where it is enough to evaluate $\mathcal{Z}(x)$ to zeroth order in $\mathcal{M}(x)$, which can be done by inserting Eq. (2) into the equation for the wave-function, which accordingly becomes

$$\left[1 + \frac{\alpha\xi}{8\pi} \right] \mathcal{Z}(x) = 1 - \frac{\alpha\xi}{4\pi} \int_x^\Lambda \frac{dy}{y} \mathcal{Z}(y).$$

The unique solution of this is [1]

$$\mathcal{Z}(x) = \frac{1}{1 + \alpha\xi/8\pi} \left(\frac{x}{\Lambda^2} \right)^\nu, \quad (4)$$

where

$$\nu = \frac{2\alpha\xi}{8\pi + \alpha\xi},$$

in agreement with lowest order perturbation theory.

On putting this solution for $\mathcal{Z}(x)$ into Eq. (3), we find the following first-order equation for $\mathcal{M}(x)$:

$$\begin{aligned} \left[1 + \frac{\alpha\xi}{8\pi} \right] \left(\frac{x}{\Lambda^2} \right)^{-\nu} \mathcal{M}(x) &= \frac{3\alpha}{8\pi} \int_0^\Lambda dy \mathcal{M}(y) \left\{ 1 + \left(\frac{y}{x} \right)^\nu + \frac{y+x}{y-x} \left[1 - \left(\frac{y}{x} \right)^\nu \right] \right\}_{x>} \frac{1}{x} \\ &\quad - \frac{3\alpha}{8\pi} \int_0^\Lambda dy \left(\frac{y}{x} \right)^\nu \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \frac{xy}{x_2^2} \\ &\quad + \frac{\alpha\xi}{4\pi} \int_0^x dy \left(\frac{y}{x} \right)^{\nu+1} \mathcal{M}(y) + \frac{\alpha\xi}{4\pi} \mathcal{M}(x) x^{-\nu} \int_x^\Lambda dy y^{\nu-1}. \end{aligned} \quad (5)$$

After evaluation of the last integral above, the term on the left-hand side cancels. This is a consequence of the renormalizability of the Curtis-Pennington approximation. The ultraviolet cut-off can now be taken to infinity, and after cancelling a factor of α throughout the equation, we find

$$\begin{aligned} \mathcal{M}(x) &= \frac{3\nu}{2\xi} \int_0^\infty \frac{dy}{x_>} \left\{ 1 + \left(\frac{y}{x} \right)^\nu + \frac{y+x}{y-x} \left[1 - \left(\frac{y}{x} \right)^\nu \right] \right\} \mathcal{M}(y) \\ &\quad - \frac{3\nu}{2\xi} \int_0^\infty dy \frac{x_<}{x_>} \left(\frac{y}{x} \right)^\nu \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} + \nu \int_0^x \frac{dy}{y} \left(\frac{y}{x} \right)^{\nu+1} \mathcal{M}(y), \end{aligned} \quad (6)$$

with $x_< = \min(x, y)$. The last equation is scaling invariant, and it is solved by

$$\mathcal{M}(x) = x^{-\alpha}, \quad (7)$$

on condition that s satisfies

$$\xi = \frac{3\nu(\nu-s+1)}{2(1-s)} \left[\frac{3\pi \cot \pi(v-s) + 2\pi \cot \pi s - \pi \cot \pi v}{\nu} + \frac{1}{\nu+1} + \frac{2}{1-s} + \frac{3}{s-\nu} + \frac{1}{s-\nu-1} \right] \quad (8)$$

and also on condition that $0 < s < 1$ and $0 < s - \nu < 1$.

In a chosen gauge specified by ξ , this equation defines roots s for any value of the coupling α . Bifurcation occurs when two of these roots [with $s \in (0, 1)$] are equal. Then $\alpha \equiv \alpha_c$. To understand how and when this happens, it is easiest to consider first the situation in the Landau gauge, $\xi = 0$ i.e. $v = 0$. Then Eq. (8) is particularly simple and has just two roots in $(0, 1)$ for each value of α . For small α these roots are real. As α is increased, they approach one another, becoming equal at criticality, when $\alpha_c = 0.93367$. The boundary conditions imposed by Eq. (5) at $x = \Lambda^2$ demand that the behaviour of the mass function be oscillatory, and that implies that the roots in Eq. (8) are complex. Thus only for α greater than α_c do Eqs. (5, 6) have a non-zero solution for $\mathcal{M}(x)$: only then can chiral symmetry breaking occur.

In other than the Landau gauge, particularly when ξ is large, Eq. (8) has more than two roots in $(0, 1)$, but we are of course interested in the ones that are continuously connected to the two that are present in the Landau gauge. A necessary condition for equality of two roots is $\partial \xi / \partial s = 0$, where $\xi(s, \nu)$ is given by Eq. (8) and the partial derivative is to be taken at constant ν . Simultaneous solution of this condition, and Eq. (8) itself, gives ξ as a function of ν , which can be more meaningfully inverted to give α_c as a function of ξ . This work cannot be performed analytically; but numerical procedures built into Maple and Mathematica have both been used to obtain $\alpha_c(\xi)$. Indeed the whole procedure can be automatized by using the FindMinimum function of Mathematica.

The results are listed in Table 2 and illustrated in Fig. 1, where we have plotted the critical α_c against ξ over the rather large domain $-5 \leq \xi \leq 15$. The most important thing to note is the reassuringly weak gauge dependence. This is in keeping with the expectations from the results of [2] at $\xi = 0, 1$ and 3. That analysis involved the numerical solution of the fully coupled equations of Table 1 on a fine mesh of values of α , followed by an

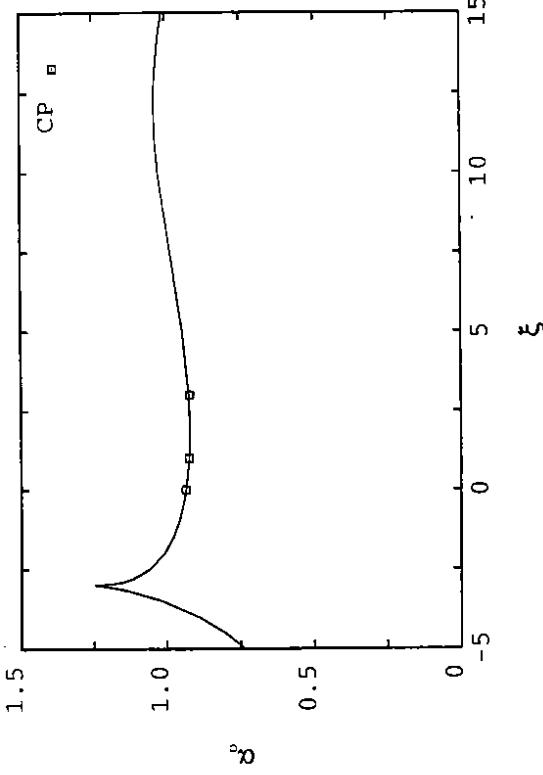


Figure 1: Critical coupling, α_c , as a function of the gauge parameter, ξ . CP labels the numerical results of [2].

extrapolation $M(x) \rightarrow 0$ to obtain the critical value α_c . Agreement with the present results (which of course can be easily obtained to many decimal places) is to several parts per mil: the results of [2] fall squarely on the curve, as can be seen in Fig. 1. At $\xi = 0$ we find $\alpha_c = 0.934$ and at $\xi = 1$ we have $\alpha_c = 0.923$. The curve has a local minimum at $\xi = 1.830$, where $\alpha_c = 0.921$. Thereafter α_c increases and at $\xi = 5$, for example, it has risen to 0.945. This variation, over rather a large range of the gauge parameter, is only a couple of percent, which indicates the vast superiority [8] of the Curtis-Pennington Ansatz over previous Ansätze for the vertex function that various people [9, 10], including one of the present authors [11], have made in the past in an attempt to improve on the ladder approximation. Thus Rembielska [9], using the gauge technique of Delbourgo, Salam and West [12], finds $\alpha_c = \pi/(3 + \xi)$. This dramatic gauge dependence can be traced to the fact that his fermion functions do not agree with perturbation theory in the weak coupling regime — a limit that should surely be respected to be physically relevant. In complete contrast, Kondo [13] finds a gauge-independent coupling, $\alpha_c = \pi/3$, but at the expense of using a vertex that has unintended singularities corresponding to the exchange of massive scalar particles — scalars not present in his original Lagrangian.

ξ	α_c	γ_m
20	0.896171	1.366598
18	0.945096	1.325094
16	0.990483	1.279108
14	1.024887	1.230017
12	1.038213	1.181923
10	1.025070	1.141526
8	0.993709	1.113341
6	0.959386	1.095898
5	0.944740	1.089788
4	0.932991	1.084596
3	0.924882	1.079703
2	0.921272	1.074401
1	0.923439	1.067732
0	0.933667	1.058069
-1	0.956804	1.041866
-2	1.006606	1.007788
-3	1.247035	0.513052
-4	0.890145	-0.073034
-5	0.732071	-0.193549

Table 2: Critical coupling, α_c , and anomalous dimension, γ_m , for a range of the gauge parameter, ξ .

Returning to our results shown in Fig. 1, for larger values of the gauge parameter, the curve turns down again, the maximum occurring at $\xi_{\text{max}} = 12.00$, with $\alpha_c(\xi_{\text{max}}) = 1.038$. For negative values of ξ , α_c first increases slowly and then has an interesting cusp at $\xi = -3$. This is brought about by the cancellation of several terms on the right hand side of Eq. (5). Thus α_c rises steeply on either side of $\xi = -3$, with $\alpha_c(-3) = 1.247$. For more negative ξ the curve descends steeply.

Strictly speaking, we should check the inequality $0 < \nu - s < 1$, for $s \in (0, 1)$, required for convergence of the various integrals in Eq. (6). This restricts ξ to the domain $-2.98 < \xi < 7.36$. Although Eq. (6) is infra-red divergent for $\xi < -2.98$, it is only superficially so. For $\alpha > \alpha_c$ this potential divergence is suppressed by terms quadratic in the mass-function: y is replaced by $y + \mathcal{M}^2(y)$ at crucial places in the denominators [14]. The solution then no longer has exactly the power form of Eq. (6), but asymptotically [$x >> \mathcal{M}^2(x)$] this behaviour is still valid, and this is all we need to make the bifurcation analysis.

For $\xi \geq 7.36$, the integrals defining $\mathcal{M}(x)$ diverge in the ultra-violet, i.e. for these gauges not only the wave-function, but also the mass-function need to be renormalized. This can be done in the standard way by first writing an equation for $\mathcal{M}(x) - \mathcal{M}(\mu^2)$, and subsequently dividing by $\mathcal{M}(\mu^2)$, to define a renormalized mass-function. This allows one to extend the domain of the gauge parameter ξ for which the equations make sense.

4 Anomalous Dimension at Criticality

In solving the bifurcation equation, we have at the same time found the exponent β of Eq. (6), and hence we have deduced the ultra-violet behaviour of the mass function $\mathcal{M}(-p^2)$. This can always be expressed in quenched theories as

$$\mathcal{M}(-p^2) \sim (-p^2)^{\gamma_m/2-1}, \quad (9)$$

in the deep Euclidean region. Here γ_m is the anomalous dimension of the $\bar{\psi}\psi$ operator, related to s by $\gamma_m = 2(1-s)$ (see Eqs. (6, 9)). The values of γ_m at criticality are listed in Table 2 and plotted in Fig. 2 for the same domain of the gauge parameter ξ as in Fig. 1. We note that, for a sizeable region, γ_m is only weakly gauge-dependent, as seen in Table 2 for $-1 < \xi < 10$. However, as $\xi \rightarrow -3$, where α_c has a cusp, γ_m decreases rapidly. Nevertheless, Fig. 2 illustrates the fact that γ_m is a little larger than unity in a neighbourhood of the Landau gauge.

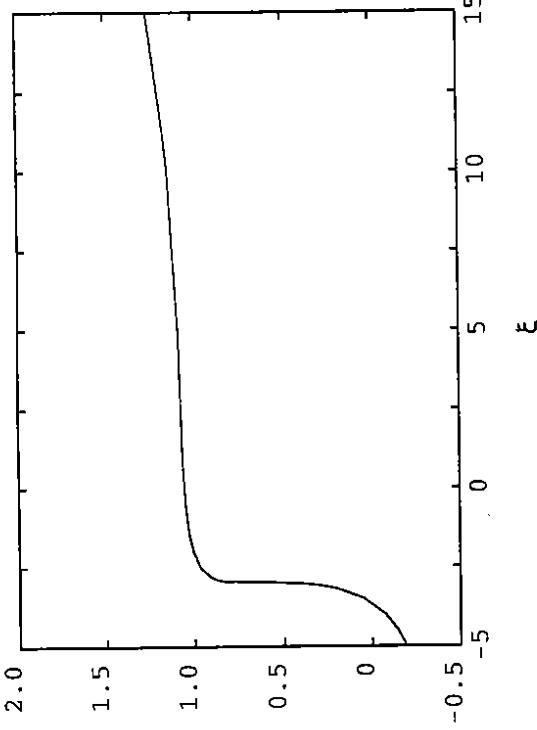


Figure 2: Anomalous dimension, γ_m , as a function of the gauge parameter, ξ . Holdom [4] claims to have constructed a general proof that $\gamma_m = 1$ in quenched QED in any gauge. So why have we obtained a different result? Let us first examine the basis of Holdom's proof. He asserts that the Dyson-Schwinger equation for the mass function can in general be written as

$$\sqrt{x} \mathcal{M}(x) = \int_{\mathcal{M}^2(0)}^{x^2} \frac{dy}{\sqrt{y}} F(x, y) \mathcal{M}(y), \quad (10)$$

where $F(x, y)$ is a function of x/y only and is symmetric under $x \leftrightarrow y$. If these conditions are true, then Holdom shows that the right-hand side of Eq. (10) must be an oscillating function of x , so that $\mathcal{M} \sim 1/\sqrt{x}$ for large x , thus proving $\gamma_m = 1$. These conditions are indeed true in the Landau gauge in the rainbow approximation, but Holdom claims to go beyond this. To obtain Eq. (10), Holdom uses the effective potential of Cornwall, Jackiw and Tomboulis [15]. His derivation [16] involves dropping certain explicit mass terms in the integrand — terms in the interaction that cannot, in fact, be dropped without violating the Ward-Takahashi identity. Thus, as already noted in [14], the equations in Table 1 allow no such simple form as Eq. (10). This is because of the appearance of the

differences $\mathcal{M}(y) - \mathcal{M}(x)$ under the loop integrals of Eqs. (5, 6). These terms are *not* specific to the Curtis-Pennington vertex, but arise solely from the requirement that the full vertex satisfy the Ward-Takahashi identity. Thus Holdom's form is just not true in general, and so it is no surprise that we find $\gamma_m \neq 1^2$.

We reiterate that the present calculation in continuum quenched QED with multiplicative renormalizability and a finite cut-off reveals an anomalous dimension just a little larger than one in a neighbourhood of the Landau gauge. This is sufficient to ensure that four-fermion operators do become relevant and QED is incomplete without them when the coupling approaches its critical value. The importance of such a result, if replicated in more general gauge theories, has been repeatedly stressed by Miransky [3, 18]. It would make the possibility of tightly bound scalar states a reality [19].

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²Lattice computations find $\gamma_m \sim 5/3$ [17]. However, as discussed in [18], the lattice implicitly includes four-fermion operators, making such work not directly comparable with the present one in the continuum.

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Curtis-Pennington Equations

$$D_F^{\mu\nu}(k) = \frac{1}{k^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) - \xi \frac{k^\mu k^\nu}{k^4}$$

$$S_F(p) = \frac{Z(-p^2)}{\gamma_\mu p^\mu - \mathcal{M}(-p^2)}$$

$$\Gamma^\mu(q, p) = \frac{1}{2} \gamma^\mu \left[\frac{1}{Z(-q^2)} + \frac{1}{Z(-p^2)} \right] + \frac{1}{2} \left\{ \frac{(q+p)^\mu \gamma^\nu (q+p)_\nu}{q^2 - p^2} + \frac{(q^2 + p^2)[\gamma^\mu (q^2 - p^2) - (q+p)^\mu \gamma^\nu (q-p)_\nu]}{[M^2(-q^2) + M^2(-p^2)]^2} \right\} \left\{ \frac{1}{Z(-q^2)} - \frac{1}{Z(-p^2)} \right\} - \frac{(q+p)^\mu}{q^2 - p^2} \left[\frac{\mathcal{M}(-q^2)}{Z(-q^2)} - \frac{\mathcal{M}(-p^2)}{Z(-p^2)} \right]$$

$$\mathcal{Z}^{-1}(x) = 1 - \frac{\alpha}{4\pi} \int_0^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} I(y, x) + \xi \frac{\alpha}{4\pi} \int_x^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} \mathcal{M}(x) \frac{Z(y)}{Z(x)}$$

$$\mathcal{Z}^{-1}(x) \mathcal{M}(x) = \frac{\alpha}{4\pi} \int_0^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} J(y, x) + \xi \frac{\alpha}{4\pi} \int_x^{\Lambda^2} \frac{dy}{y + \mathcal{M}^2(y)} \mathcal{M}(x) \frac{Z(y)}{Z(x)}$$

$$I(y, x) = \frac{3}{2} \frac{1}{y-x} \left\{ \mathcal{M}(y) \left[\mathcal{M}(y) - \frac{Z(y)}{Z(x)} \mathcal{M}(x) \right] + \frac{1}{2} \frac{(y+x)[\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2}{(y-x)^2 + [\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2} \left[1 - \frac{Z(y)}{Z(x)} \right] \right\} \left[\frac{y^2}{x^2} \theta(x-y) + \theta(y-x) \right] - \xi \left\{ 1 + \frac{\mathcal{M}(y)}{y} \left[\mathcal{M}(y) - \frac{Z(y)}{Z(x)} \mathcal{M}(x) \right] \right\} \frac{y^2}{x^2} \theta(x-y)$$

$$J(y, x) = \frac{3}{2} \mathcal{M}(y) \left\{ 1 + \frac{Z(y)}{Z(x)} + \frac{y^2 - x^2}{(y-x)^2 + [\mathcal{M}^2(y) + \mathcal{M}^2(x)]^2} \left[1 - \frac{Z(y)}{Z(x)} \right] \right\} \left[\frac{y}{x} \theta(x-y) + \theta(y-x) \right] - \frac{3x}{2} \frac{Z(y)}{Z(x)} \frac{\mathcal{M}(y) - \mathcal{M}(x)}{y-x} \left[\frac{y^2}{x^2} \theta(x-y) + \theta(y-x) \right] + \xi \frac{Z(y)}{Z(x)} \mathcal{M}(y) \frac{y}{x} \theta(x-y)$$

Table 1